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A Comparision Between Famous Discrete State Space Models

Prashant K. Shah

Electronics Engineering Department, Sardar Vallabhbhai National Institute of Technology, Surat, Gujarat, India

***Abstract*—n-D system has been very interesting areas from several past decades, due to its various applications. These n-D systems are not easy to represent compare to 1D system. There are several methods that has been used to overcome any problem but among them state space model based methods has accurate solution compare to others. In past few years, Two-dimensional (2-D) discrete Systems have attracted due to their effectiveness in describing systems in several modern engineering fields like image data processing and transformation, thermal processes, water stream heating, etc, out of available space state models.This paper review and compare different 2-D state space models. From comparative analysis we are going to show which model is best usable for which application. And also we have done stability analysis by using our derived criteria and try to conclude the best model in terms of stability.**

***Indexed Terms* —Models; Two-dimensional; Control; State-Space; Givone-Roesser(GR); Fornasni-Marchesini (FM);**

I. INTRODUCTION

State Space methods are widely used in the study of single-dimensional scalar and multivariable systems. The procedure involved will provide insight into the internal interaction of the system which is under consideration, and enables important structural information, stability, and response predictions to be established while accomplishing an efficient computational form for intended purpose.

Indeed, many computer-aided control system design packages are based entirely on state-space techniques where numerical operations on matrix number arrays are mainly required. State-space descriptions are effective in different kinds of analysis like, in transient and frequency response calculations, stability assessment, model reduction, and conformal mapping techniques.

The possibility of employing state-space realization for multidimensional control systems is obviously worth investigating as the great efficiency and flexibility in the approach and the substantial computational advantages it offers.

Nowadays, 2-D Systems has its demands in various applications in important area like image processing, signal processing, multi-dimension

Filtering, thermal processing, gas absorption, water steam heating, seismographic data processing [1]. There are several methods available which used to represent operation involved in 2-D systems. State Space model gives best utility in analysis as well as formulation of linear 2d systems, we can use non-linear systems also in state space models, using filters such as Kalman filters (EKF), and unscented Kalman filters (UKF).

During last three four decade years many authors like Attasi, Fornasini-Marchesini, Rosser, Givone-Rosser, have proposed different state space models. Out of which Fornasini-Marchesini (FM) models and Givone-Rosser models are very basic models. Among both model R and FM-2 are best model that we will see in our paper.

Rest of the paper will proceed as follows. Section II representation of basic State Space systems is done. Section III presents the famous State-Space models in detail. In Section IV we are doing comparison between these models. Section V shows the Conclusion of this paper followed by the reference section.

II. STATE SPACE SYSTEMS REPRESENTATION

State Space representation of a physical system is a mathematical model that can be represented as a set of state variables, input and output related to the 1st order differential equation [2].

State space system is the concepts of the state of dynamic system refer to a minimum set of variables which fully describe the system and its output to any given set of inputs.

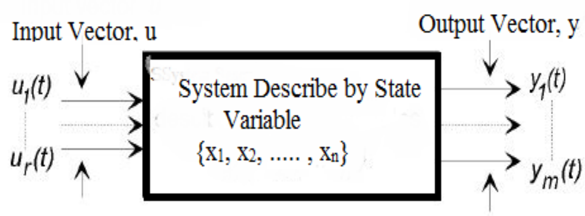

Figure 1: System inputs and outputs.

Knowledge of state space variable ($x_1, x_2 \ldots x_n$), and the input u(t) of a state determined system, is sufficient to the determine all future behaviour of it.

These are equations which have been used throughout system dynamics. These first order differential equations know as state equations, where time derivative of each state variable is expressed as state variable $x_1(t)\ldots x_n(t)$ and the system inputs $u_1(t)$ $\ldots u_r(t)$ in the general form of the n state equation is below.

$$\dot{x}_1 = f_1(x,u,t)$$

$$\dot{x}_2 = f_2(x,u,t)$$

$$\vdots \quad \vdots \qquad \ldots (1)$$

$$\dot{x}_n = f_n(x,u,t)$$

Where $x_i = \frac{dx_i}{dt}$ and $(i = 1,\ldots,n)$ for $fi\ (\mathbf{x},\mathbf{u},t)$, it can be general nonlinear system or time varying function of state variable.

The differential equation for any network can be written as,

$$\dot{x}(t) = A\,x(t) + B\,u(t) \qquad (2)$$

$$y(t) = C\,x(t) + D\,u(t) \qquad (3)$$

State equation of any system is represented by the first order differential equation as given in (2) where the vector $\vec{x}(t)$ is the state vector, and $\vec{u}(t)$ is the input vector. Equation (3) is referred to as the output equation where, the state matrix is given by A, Input matrix by B, output matrix by C, and D is the direct transition matrix.

Advantage of the state space method is that the digital and analogue computation methods form a lends itself easily to the solution of it. Further, the analysis of nonlinear systems can also be done by extending the state space method .It is well known that, it is very difficult to obtain a minimal state-space realization in the 2D case except for some special categories of 2D systems unlike the one-dimensional (1-D) case. So, it is desirable to obtain a state-space realization with as low order as possible, for a general 2-D transfer function or transfer matrix.

III. FAMOUS STATE-SPACE MODELS

There are several state space models are developed among them Attasi, Fornasini-Marchesini, Givone-Roesser, Roesser model are the famous models for 2-D state space for system analysis, and solving problems of controllability and stability like parameters.

A. *Attasi Model*

The attasi model is useful for image processing where it belongs to a large class of double indexed sequences. It is the first model Described for the 2-D State space modelling and also consider as a special case of model given by Fornasini and Marchesini. A 2-D Attasi model [3] is described by the equations,

$$x_{i+1,j+1} = -A_1A_2x_{i+1} + A_1x_{i+1,j} + A_2x_{i,j+1} + Bu_{ij}$$

$$y_{ij} = Cx_{ij} + Du_{ij} \qquad (4)$$

Where $i,j\epsilon Z_+$ (the set of non-negative integers). Here $x(i,j)\epsilon R^n$is the local state vector, $u(i,j)\epsilon R^m$ is the input vector, $y(i,j)\epsilon R^p$and is the output vector, and A_1, A_2, B, C, D are real matrices. The model was introduced by Attasi in “Systems laniaries homogeneous a deux indices,”.

B. *Fornasini-Marchesini Model*

There are two FM models are available, these models were developed by Ettore Fornasini and Giovanni Marchesini. Among two models one known as FM-1 and another known as FM-2 or FM second model. Their main goal was to introduce state space model for 2-D filter. Both models are very famous for deriving stability and other criteria but among both FM-2 models are best and famous because it is very general model we can imbedded any model in to the FM-2, Whereas FM1 has failed to some extent [5].

FMs Models were mainly developed for realization of 2D spatial filters FMs models can be used for stability purpose also, so these models are most suitable for proving stability criteria for available 2D system. There can be having various applications where it uses because it is most general model.

a) FM-1 Model

This model was introduced in 1976, in that they took problem of 2-D filter and input-output of such system is represented by power series in two variables and nerode space is generated, state space is general and infinite dimensional. For rational power series the dynamics of the filter is described by updating equations on finite-dimensional local state space.

By this model they are first to realised the difference between 1D and 2D system. They have given global state and local state concept for 2D system. Where global state is infinite in dimension that preserve all the past information and local state gives recursion to perform each step in 2D system [4]. However, Fornasini and Marchesini failed to exploit fully the structure of the global state’s relation to the local state, so it is the unsatisfactory state-space model that they introduced [4].

FM-1 model system can be given by the following figure,

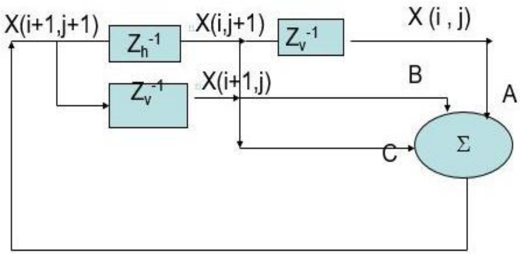

Fig. FM-1 System

With modeling equation,

$$x_{i+1,j+1} = A_0 x_{i+1,j} + A_1 x_{i+1,j} + A_2 x_{i,j+1} + Bu_{ij}$$

$$y_{ij} = Cx_{ij} + Du_{ij} \quad (5)$$

where, $i, j\epsilon Z_+$ (The set of nonnegative integers) here $x_{ij}\epsilon R^n$ is the local state vector, $u_{ij}\epsilon R^m$ is the input vector, $y_{ij}\epsilon R^p$ is the output vector, $A_k(k = 0,1,2), C, D$ are real matrices [1].

There are also certain advantages of the first FM model i.e., the well-known Attasi state-space model have some special structural advantages for first FM Model. The Attasi model can't be realized using FM-2 model. It can only be realized using the first FM model. But there is still some advantage of using FM-2 model mostly as compare to FM-1 that we will see in next section.

b) FM-2 Model

This model was introduced because FM1 model was not generic to do analysis for all the T.F or differential equations. So in 1978, they introduced FM2 model which is based on FM1 but they have given new model which is general in use. They extended FM1 concepts of local reachability and observe ability and their properties.

Then the definition of internal stability will be naturally introduced and we shall develop a stability criterion and connections between internal and external stability. They conclude that we can embed any model in this model. Any causal system (not strictly causal) can be embedded in this model.

FM2 model uses modelling equation,

$$x(i+1, j+1) = A_1 x(i, j+1) + A_2 x(i+1, j) + B_1 u(i, j+1) + B_2 u(i+1, j)$$

$$z(i, j) = Cx(i, j) + Du(i, j) \quad (6)$$

where$i, j\epsilon Z_+, x(i, j)\epsilon R^n$is the local state vector,$u(i, j)\epsilon R^m$ is the input vector, $z(i, j)\epsilon R^p$ is the output vector, $A_k(k = 1,2), C, D$are real matrices [5].

FM-2 model is more satisfactory in terms of Stability as compare to the FM-1 model [6]. It also a more general model as compare to FM-1.

C. GIVONE-ROESSER MODEL

This state space model was developed by Donald D. Givone and Robert P. Roesser in 1972. This model was developed fortwo-dimensional linear iterative circuits. Linear Iterative circuits are similar in nature to sequential machines with an added generality of existing over more than one dimension. Most of the industrial applications are involved in same executing operation for many times over a fixed interval, when each operation is complete. It will reset the staring location takes places and go for next operation. These types of applications are Iterative system; they are widely using in robotics, hard disk drives, chemical batch process, and urban traffic systems.

The GR model is given by,

$$\begin{pmatrix} x^h(i+1, j) \\ x^v(i, j+1) \end{pmatrix} = \begin{pmatrix} A_1(i,j) & A_2(i,j) \\ A_3(i,j) & A_4(i,j) \end{pmatrix} \begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + \begin{pmatrix} B_1(i,j) \\ B_2(i,j) \end{pmatrix} \mathrm{u(i,j)} \quad (7)$$

Where x is local state, x^h an n-vector is horizontal state, x^v an m-vector is vertical state. $u(i, j) \epsilon R^m$ is the input vector, and A1, A2, A3, A4, B1, B2, are real matrices [7].

As per Givone-Roesser (GR) model sufficient condition for 2-D system stability is,

$$\begin{pmatrix} P_1 & 0 \\ 0 & P_2 \end{pmatrix} - A^T(P_1 + P_2)A > 0, \quad (8)$$

Givone-Roesser is the most satisfactory in that respect; it is also the most general since the Attasi and Fomasini-Marchesini models can be imbedded in the Givone-Roesser model [5].

D. ROESSER MODEL

This model was introduced in 1975 by ROBERT P. ROESSER, it is developed for application of image processing to overcome various problems in image processing and analysis of image. There are several ways has been used for representation of image processing, like partial differential equations, transfer function, convolution summations.

This ways can be help to solve problem of image processing. The two-dimensional Fourier transform is given by the transfer function which relates an output image to that of the input image. Complex optical systems are easily described by combinations of transfer functions that correspond to individual components of the optical system i.e. partial differential equations used by habib to describe a model estimation of noise corrupted images by this model correspond extension to kamal filter. This time-discrete state-space model offers great utility in the formulation and analysis of systems for linear image processing [8].

Linear system that describe by T.F and convolution summation or partial differential equations are easily formulated in state space model. Once formulate many know technique can be apply to that model for analysis. He developed a discrete model of time-discrete systems that closely parallels the well-known state space model for linear image processing. Because of this parallelism many known concepts of the temporal model may be carried over to the spatial model, which is done by generalizing two coordinates in space from a single coordinate of time. The spatial model will hopefully have some of the same utility that of temporal model for one-dimensional linear systems in unifying the study of two-dimensional linear systems [1].

Roesser model ismodel by following equations,

$$x(i,j) = \begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} \quad (9)$$

where x is local state, x^h an n-vector is horizontal state, x^v an m-vector is vertical state.

$$\begin{pmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{pmatrix} = \begin{pmatrix} A_1(i,j) & A_2(i,j) \\ A_3(i,j) & A_4(i,j) \end{pmatrix} \begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + \begin{pmatrix} B_1(i,j) \\ B_2(i,j) \end{pmatrix} u(i,j) \quad (10)$$

$$y(i,j) = (C_1\ C_2) \begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + Du(i,j). \quad (11)$$

$u(i,j) \epsilon R^m$ is the input vector, $y(i,j) \epsilon R^p$ is the output vector and A1, A2, A3, A4, B1, B2, C, D are real matrices [5].

Roesser model is most general model in 2-D state space model. [5] Proved that model like FM and Attasi can be embedded in the Roesser model so there is not loss of generality. Rosser model has local state which is divided into horizontal and vertical state which gives the size of recursions to be perform each step. FIR, IIR can also be realized using Rosser model [9].

IV. COMPARISION

So, Based on our Study of different State Space model now we will see the general comparison of these models and we will also see the comparison based on their use, application, and soon, [10,11]. The Comparison is telling us about the basic Difference between these models and it also tells us that which model is best suitable for which application. So, below table will gives us the idea about the basic difference between these models.

Table 1. General Comparisons

FM-1	FM-2	GR	R
Uses the algebraic point of view of Nerode equivalence.	Uses the algebraic point of view of Nerode equivalence.	Uses Circuit approach to solve the problems	Uses Circuit approach to solve the problems
Introduce a Global state and local state in the 2-D case. (Global state is infinite dimension and store past information,)	It also Introduce a Global state and local state in the 2-D case [12].	Has local state which is divided into horizontal and vertical sate. (Local state gives the size of recursions to be perform each step)	Has local state which is divided in to horizontal and vertical state.
Not a general model(Because not including all the causal TFs)	Most General models(Can embedded other models in This)	Not a general model	Most General models(Can embedded other models in This)
The succeeding state (i+1,j+1) depends on state (i,j),(i,j+1) and (i+1,j).	The succeeding state (i+1,j+1) depends on state (i,j+1) and (i+1,j).	--	--

Table I is analysis of models in general forms. In point of stability we can differentiate stability in as table II for FM2 and III and IV is comparison of for GR model with existing and proposed model. Where stability criteria is discussed based on computational complexity, When we use Matlab LMI toolbox for solving derived stability criteria, at that time computational complexity of stability criteria means how much effort has been taken by processor to solve it.. i.e. suppose if we have derived two stability criteria and also we have taken stable system to check it, if the Matlab LMI result shows for Table III and IV shows existing and proposed method comparison in that we have only two stability criteria which give more efficient result than existing.

Table 2. Stability Analysis of Fm1 Model [6]

	Proposed Method Result(I)	**Proposed Method Result(II)**	**Comment**
Stability criteria	$\begin{pmatrix} -P & PA & 0 & PH \\ A^T P & -Q & \varepsilon E^T & 0 \\ 0 & \varepsilon E & -\varepsilon I & 0 \\ H^T P & 0 & 0 & -\varepsilon I \end{pmatrix} > 0$	$\begin{pmatrix} P_1 - \varepsilon_1 E_1^T E_1 & 0 & A_1^T P & PH_1 & PH_2 \\ 0 & P_1 - \varepsilon_1 E_1^T E_1 & A_2^T P & 0 & 0 \\ PA_1 & PA_1 & P & 0 & 0 \\ H_1^T P & 0 & 0 & \varepsilon_1 I & 0 \\ H_2^T P & 0 & 0 & 0 & \varepsilon_2 I \end{pmatrix} > 0$	Existing Result is not available
Computational complexity, memory	Less	More	Result -I is better (please ,refer next slide)
Speed	More	less	
Gap Between Necessary and sufficient conditions	More	Less	Result-II is better
Ease of handling algorithm for stability criteria	yes	yes	Better due to Matlab LMI toolbox

Table 3. Existing Result [14]

NUMERICAL RESULTS.

Ex\Thm	Cor 1 suff cond.	Thm 5 suff cond.	Thm 6 suff cond.	Thm 2 suff cond.	Thm 4 nece cond.	Thm 3 suff cond.	Conclusion
1	(3) 0.63						Stable
2	(2) 1.10 (3) 1.40 (4) 0.924						Stable
3	(2) 1.08 (3) 1.17 (4) 1.14	(4) 1.270	(2) 0.821				Stable
4	(2) 1.17 (3) 1.10 (4) 1.512	(4) 1.858	(2) 1.23 (3) 2.799 (4) 1.492	0.946			Stable
5	(2) 5.60 (3) 4.40 (4) 6.245	(4) 6.729	(2) 6.043 (3) 11.91 (4) 6.195	3.567	1.123		Unstable
6	(2) 2.85 (3) 2.40 (4) 2.51	(4) 2.889	(2) 2.149 (3) 4.696 (4) 2.253	1.267	0.259	0.742	Stable

Table 4. Our Results

Ex\Thm	Thm 1	Thm 2	Conclusion
1	√	-	stable
2	×	√	stable
3	×	√	stable
4	√	-	stable
5	-	-	unstable
6	×	√	stable

Table 5. Stability Analysis [7]

	Proposed Method(I)	Proposed Method(II)	Existing Methods	comment
Stability Criteria	$\begin{pmatrix} -P & PA & 0 & PH \\ A^TP & -P & kE^T & 0 \\ 0 & kE & -kI & 0 \\ H^TP & 0 & 0 & -kI \end{pmatrix} > 0$	$\begin{pmatrix} P_1-\varepsilon_1E_1^TE_1-\varepsilon_3E_3^TE_3 & 0 & A_1^TP_1 & A_3^TP_2 & 0 & 0 & 0 & 0 \\ 0 & P_2-\varepsilon_2E_2^TE_2-\varepsilon_4E_4^TE_4 & A_1^TP_1 & A_4^TP_2 & 0 & 0 & 0 & 0 \\ P_1A_1 & P_1A_2 & P & 0 & P_1H_1 & P_1H_2 & 0 & 0 \\ P_2A_3 & P_2A_4 & 0 & P_2 & 0 & 0 & P_2H_3 & P_2H_4 \\ 0 & 0 & H_1^TP_1 & 0 & \varepsilon_1I & 0 & 0 & 0 \\ 0 & 0 & H_2^TP_1 & 0 & 0 & \varepsilon_2I & 0 & 0 \\ 0 & 0 & 0 & H_3^TP_2 & 0 & 0 & \varepsilon_3I & 0 \\ 0 & 0 & 0 & H_4^TP_2 & 0 & 0 & 0 & \varepsilon_4I \end{pmatrix} > 0$	Total six result	Proposed method is LMI based, existing method is linear algebra based
Computational complexity, memory	Less	More	Much more(Total six algorithm)	Existing Method required to six algorithms in contrast to only two for proposed method, in terms of memory Result –I is better
Speed	More	Less		
Gap between necessary and sufficient Conditions	More	Less	Much more	Purposed method result-II is better than result-I and existing method
Ease of handling algorithm for stability criteria	Yes	Yes	No	Proposed methods use LMI approach which helps to get fast and accurate result

Table III and IV shows existing and proposed method comparison in that we have only two stability criteria which give more efficient result than existing.

Stability analysis example:

Borrowed example of [15] LSIV 2-D Attasi's model filter (PSV system) using circulant matrices to test our derived stability criteria.

$$\overline{A_1} = \begin{pmatrix} 0.5 & -0.5 & 0.125 & -0.125 \\ -0.125 & 0.5 & -0.5 & 0.125 \\ 0.125 & -0.125 & 0.5 & -0.5 \\ -0.5 & 0.125 & -0.125 & 0.5 \end{pmatrix},$$

$$\overline{A_2} = \begin{pmatrix} 0.5 & 0 & -0.015 & 0.25 \\ 0.25 & 0.5 & 0 & -0.015 \\ -0.015 & 0.25 & 0.5 & 0 \\ 0 & -0.015 & 0.25 & 0.5 \end{pmatrix},$$

$$\bar{B} = (1 \quad 0.39 \quad -1 \quad 0.45)^T,$$

$$\bar{C} = (1 \quad -1 \quad -1 \quad 1),$$

$$\boldsymbol{A_1} = \overline{\boldsymbol{A_1}} \ \ and\ C = \bar{C}$$

$$A_2(0,0) = \begin{pmatrix} 0.5 & 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & -0.5 & 0.5 \end{pmatrix},$$

$$A_2(0,1) = \begin{pmatrix} 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0.5 \end{pmatrix},$$

$$B(0,0) = (-1 \quad -1 \quad -0.25 \quad -1)^T$$

$$B(0,1) = (1 \quad -0.25 \quad -1 \quad -0.5)^T$$

$$A_0 = A_1A_2.$$

Period of PSV in this case is 1:2.

for FM2:

$$A = (A_1 \quad A_2),$$

where $A_1 = \overline{A_1}$,

$$A_2(0,0) = \begin{pmatrix} 0.5 & 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0.5 & 0.5 \\ 0 & -0.5 & 0.5 & 0.5 \\ 0.5 & 0 & -0.5 & 0.5 \end{pmatrix}, A_2(0,1) = \begin{pmatrix} 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 \\ 0 & 0.5 & 0.5 & 0.5 \end{pmatrix},$$

$$\Delta A = A(A_1, A_2(0,0)) - A(A_1, A_2(0,1));$$

$$\Delta A = HFE.$$

$$\Delta A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 & 0 \end{pmatrix},$$

after getting ,H F, E we can follow previous steps for stability analysis.

for FM1:

$$A = (A_0 \quad A_1 \quad A_2),$$

for R:

$$A = \begin{pmatrix} A_{00} & A_{01} \\ A_{10} & A_{11} \end{pmatrix},$$

values are available for FM1 and for FM2 as shown previously, to convert FM1 matrices values in R Form we have taken reference of [17] and converted in suitable matrices form. FM1 and R are not independent and can be mutually recasted.

FM1

$$E'x(i+1,j+1) = A_1x(i+1,j+1) + A_2x(i+1,j) + A_0(i,j) + Bu(i,j) + B_1u(i,j+1) + B_2u(i+1,j),$$

R

$$E\begin{pmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{pmatrix} = A\begin{pmatrix} x^h(i,j) \\ x^v(i,j) \end{pmatrix} + Bu(i,j),$$

where $E = I_N$, $N = t_1 + t_2$.

To convert FM1 in to R model we are assuming horizontal vector $\xi(i,j) = E'x(i,j+1) - A_1x(i,j)$.

and equation of FM1 in R form would be

$$\begin{pmatrix} I_n & -A_2 \\ 0 & E' \end{pmatrix}\begin{pmatrix} \xi(i+1,j) \\ x(i,j+1) \end{pmatrix} = \begin{pmatrix} 0 & A_3 \\ I_n & A_1 \end{pmatrix}\begin{pmatrix} \xi(i,j) \\ x(i,j) \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix}u(i,j)$$

taking this concept we get matrix 'A' for R which is,

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0.5000 & 0 & -0.0150 & 0.2 \\ 0 & 0 & 0 & 0 & 0.2500 & 0.5000 & 0 & -0.0 \\ 0 & 0 & 0 & 0 & -0.0150 & 0.2500 & 0.5000 & \\ 0 & 0 & 0 & 0 & 0 & -0.0150 & 0.2500 & 0.5 \\ 1.0000 & 0 & 0 & 0 & 0.7500 & -0.5000 & 0.1231 & -0.1 \\ 0 & 1.0000 & 0 & 0 & -0.1563 & 0.7500 & -0.5000 & 0.1 \\ 0 & 0 & 1.0000 & 0 & 0.1231 & -0.1563 & 0.7500 & -0.5 \\ 0 & 0 & 0 & 1.0000 & -0.5000 & 0.1231 & -0.1563 & 0.7 \end{pmatrix}$$

using these matrices for FM1 ,FM2 and R stability analysis has been done using previous steps available in[16] for PSV system.

Table 6. Stability Analysis For Above Example

	FM1		FM2		R	
	TH1	TH2	TH1	TH2	TH1	TH2
Stability condition	×	√	√	-	√	-

Such types of analysis have been done for check stability criteria using our derived criteria.

V. CONCLUSION

This Paper Studied different state space models with its basics. All the famous state space models are discussed with its equations. The major difference between famous state space models has been tried to conclude, using stability analysis also we checked that which model is best and drawback of some model was discussed. How it can be applicable for n-D system it was discussed. And we conclude that FM2 and R model is best in terms of stability the ideas of implementation on physical system of these models is discussed. Various latest applications where this model has been used and some criteria on which this models depends for performing various task properly is try to give.

References

[1] T. Kaczorek, Two-Dimensional Linear Systems, Springer, Berlin, 1985.

[2] Derek Rowell" State-Space Representation of LTI Systems" 2.14 Analysis and Design of Feedback Control Systems.

[3] Attasi in "Systemes lineaires homogenes a deux indices," *IRIA Rapport Laboria*, No.

[4] S. Y. Kung, B. Levy, M. Morf, T. Kailath, "New results in 2-d systems theory part ii: 2-d state-space models; realization and the notions of controllability' observability and minimality", *Proceedings of the IEEE*, vol. 65, no. 6, pp. 945-961, June 1977.

[5] E. Fornasini, G. Marchesini, "Doubly indexed dynamical systems: Statespace models and structural properties," *Math. Syst. Theory*, vol. 12, pp.59-72, 1978.

[6] Prashant k shah,' LMI based Stability criteria for 2-D PSV system described by FM-2 Model,submitted

[7] Prashant k shah,'Stability analysis of 2-D PSV system represented by Givone–Roesser Model',submitted

[8] Dr.(Mrs.) U. D. Dalal, Dr. S. N. sharma & Prof. Prashant K. shah, "Stability analysis of 2-D PSV system represented by Givone–Roesser Model", " Department of Electronics Engineering, SVNIT ,Surat chapter, IETE-2015, Gujarat, India.

[9] R. P. Roesser, "A discrete state-space model for linear image processing," *IEEE Trans. Automat. Control*, vol. 20, pp. 1-10, 1975.

[10] Eric Rogers'The Fornasini-Marchesini Model — its Role in the Analysis and Control of Physical Systems'

[11] Manish Tiwari, Amit Dhawan"A Survey on the Stability of 2-D Discrete Systems Described by Fornasini-Marchesini Second Model"*Control Circuits and Systems*, January 2012, 3, 17-22 http://dx.doi.org/10.4236/cs.2012.31003 Published Online January 2012.

[12] S. Y. Kung, B. C. Levy, M. Morf, and T. Kailath,"New results in 2-D systems theory, Part II: 2-Dstate-space models – realization and the notionsof controllability, observability, and minimality,"*Proc. IEEE*, vol. 65, pp. 945-961, 1977.

[13] M. Bisiacco, E. Fornasini, and G. Marchesini, "Dynamic regulation of 2D Systems: A statespace approach," *Linear Algebra and Its Applications*, vol. 122/123/124, pp.195-218, 1989.

[14] Tamal Bose, Mei-Qin Chen, and Ratchaneekorn Thamvichai,' Stability of the 2-D Givone–Roesser Model With Periodic Coefficients', IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: REGULAR PAPERS, VOL. 54, NO. 3, MARCH 2007.

[15] R. Thamvichai, T. Bose, D. M. Etter , "2-D periodically shift variant filters without multipliers", *Proc. World Multiconf. Systemics Cybernetics and Informatics*, vol. 1, 2000-July.

[16] Prashant Shah ,Prof. Haranath Kar, Prof.Vinaya Kumar Srivastava, "Stability Analysis of 2-D PSV Filters described by FM-1 Model" ICECE-2011 IEEE conference at Yi-Cheng, China.

[17] K. Galkowski, "The Fornasini-Marchesini and the Roesser model: Algebraic methods for recasting", *IEEE Trans. Automat. Contr.*, vol. 41, pp. 107-112, Jan. 1996.