

Newton Forward and Backward Interpolation Method

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Abstract- This paper gives us the basic information of “Newton forward and backward interpolation method”. These methods are used to solve problem on newton interpolation by forward or backward interpolation method. For different problem we have different method, this is explained by solving problem below on both newton forward and backward interpolation method. We are also deriving formula for newton forward interpolation method and newton backward interpolation method.

I. INTRODUCTION

- Interpolation is the method of finding the value of a function for any intermediate value for independent variable, while the process of calculating the value of the function outside the given range is called extrapolation. [4]
- Forward Differences: The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2 \dots y_n - y_{n-1}$ when denoted by $dy_0, dy_1, dy_2 \dots dy_{n-1}$ are respectively, called the first forward differences.

Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0	Δy_0				
x_1 (= $x_0 + h$)	y_1	Δy_1	$\Delta^2 y_0$	$\Delta^3 y_0$		
x_2 (= $x_0 + 2h$)	y_2	Δy_2	$\Delta^2 y_1$	$\Delta^3 y_1$	$\Delta^4 y_0$	
x_3 (= $x_0 + 3h$)	y_3	Δy_3	$\Delta^2 y_2$	$\Delta^3 y_2$	$\Delta^4 y_1$	$\Delta^5 y_0$
x_4 (= $x_0 + 4h$)	y_4	Δy_4	$\Delta^2 y_3$			
x_5 (= $x_0 + 5h$)	y_5					

II. NEWTON’S GREGORY FORWARD INTERPOLATION METHOD

This formula is mainly useful for interpolating the values of $f(x)$ near the starting of the set of values given. “h” is the common difference and $u = (x - a) / h$, here “a” is first term.

Example: -

Input: - value of sin 52

a	45°	50°	55°	60°
a°	0.7071	0.7660	0.8192	0.8660

Output:

x°	Differences			
	$10^1 y$	$10^1 \Delta y$	$10^1 \Delta^2 y$	$10^1 \Delta^3 y$
45°	7071	589	-57	
50°	7660	532	-64	-7
55°	8192	468		
60°	8660			

Value at Sin 52 is 0.788003

Below is the implementation of newton forward interpolation method.

```

CPP Program to interpolate using
newton forward interpolation
#include <bits/stdc++.h>
using namespace std;
// calculating u mentioned in the formula
float u_cal(float u, int n)
{
    float temp = u;
    for (int i = 1; i < n; i++)
        temp = temp * (u - i);
    return temp;
}
// calculating factorial of given number n
int fact(int n)
{
    int f = 1;
    for (int i = 2; i <= n; i++)
        f *= i;
    return f;
}
int main()
    
```

```

{
// Number of values given
int n = 4;
float x[] = { 45, 50, 55, 60 };
// y[][] is used for difference table
// with y[][0] used for input
float y[n][n];
y[0][0] = 0.7071;
y[1][0] = 0.7660;
y[2][0] = 0.8192;
y[3][0] = 0.8660;
// Calculating the forward difference
// table
for (int i = 1; i < n; i++) {
    for (int j = 0; j < n - i; j++)
        y[j][i] = y[j + 1][i - 1] - y[j][i - 1];
}
// Displaying the forward difference table
for (int i = 0; i < n; i++) {
    cout << setw(4) << x[i]
        << "\t";
    for (int j = 0; j < n - i; j++)
        cout << setw(4) << y[i][j]
            << "\t";
    cout << endl;
}
// Value to interpolate at
float value = 52;
// initializing u and sum
float sum = y[0][0];
float u = (value - x[0]) / (x[1] - x[0]);
for (int i = 1; i < n; i++) {
    sum = sum + (u_cal(u, i) * y[0][i]) /
        fact(i);
}
cout << "\n Value at " << value << " is "
    << sum << endl;
return 0;
}
    
```

Output

45	0.7071	0.0589	-	-
50	0.766	0.0532	-	
55	0.8192	0.0468		
60	0.866			

Value at 52 is 0.788003

• Backward Differences

The differences $y_1 - y_0, y_2 - y_1, \dots, y_n - y_{n-1}$ when denoted by dy_1, dy_2, \dots, dy_n , respectively, are named as first backward difference. Thus the first backward differences are :

Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
x_1 (= $x_0 + h$)	y_1	∇y_1				
x_2 (= $x_0 + 2h$)	y_2	∇y_2	$\nabla^2 y_2$			
x_3 (= $x_0 + 3h$)	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$		
x_4 (= $x_0 + 4h$)	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$	
x_5 (= $x_0 + 5h$)	y_5	∇y_5	$\nabla^2 y_5$	$\nabla^3 y_5$	$\nabla^4 y_5$	$\nabla^5 y_5$

III. NEWTON'S GREGORY BACKWARD INTERPOLATION FORMULA

This formula is used when the value of $f(x)$ is required at the end of the table. h is known as the common difference and $u = (x - x_n) / h$, Here x_n is last term in table.^[3]

Example:Input: Population in 1925

Year(x):	1891	1901	1911	1921	1931
Population(y): (in thousands)	46	66	81	93	101

Output:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1891	46				
1901	66	20			
1911	81	15	-5		
1921	93	12	-3	2	
1931	101	8	-5	3	-1

Value in 1925 is 96.8368

Below is the implementation of newton backward interpolation method.

```

// CPP Program to interpolate using
// newton backward interpolation
#include <bits/stdc++.h>
    
```

```
using namespace std;
// Calculation of u mentioned in formula
float u_cal(float u, int n)
{
    float temp = u;
    for (int i = 1; i < n; i++)
        temp = temp * (u + i);
    return temp;
}
// Calculating factorial of given n
int fact(int n)
{
    int f = 1;
    for (int i = 2; i <= n; i++)
        f *= i;
    return f;
}
int main()
{
    // number of values given
    int n = 5;
    float x[] = { 1891, 1901, 1911,
                 1921, 1931 };
    // y[][] is used for difference
    // table and y[][0] used for input
    float y[n][n];
    y[0][0] = 46;
    y[1][0] = 66;
    y[2][0] = 81;
    y[3][0] = 93;
    y[4][0] = 101;
    // Calculating the backward difference table
    for (int i = 1; i < n; i++) {
        for (int j = n - 1; j >= i; j--)
            y[j][i] = y[j][i - 1] - y[j - 1][i - 1];
    }
    // Displaying the backward difference table
    for (int i = 0; i < n; i++) {
        for (int j = 0; j <= i; j++)
            cout << setw(4) << y[i][j]
                << "\t";
        cout << endl;
    }
    // Value to interpolate at
    float value = 1925;
    // Initializing u and sum
    float sum = y[n - 1][0];
    float u = (value - x[n - 1]) / (x[1] - x[0]);
    for (int i = 1; i < n; i++) {
```

```
        sum = sum + (u_cal(u, i) * y[n - 1][i]) /
                    fact(i);
    }
    cout << "\n Value at " << value << " is "
        << sum << endl;
    return 0;
}
```

Output:

46				
66	20			
81	15	-5		
93	12	-3	2	
101	8	-4	-1	-3

Value at 1925 is 96.8368

IV. USE OF NEWTON FORWARD AND BACKWARD INTERPOLATION METHOD

Forward interpolation formula is used to interpolate the values of y nearer to the beginning value of the given table. Also this formula is applicable if in case where h (difference in in travel) is constant.

Now coming to backward difference formula [1]

This formula is used to interpolate the values of y nearer to the end of the table (or) tabular values.

These are the places, where we need to apply those formulas.

These formulas are used when the in travels for computing a value of a function are equidistant.

CONCLUSION

The degree of the polynomial is one less than the number of pairs of observations. The polynomial that represents the given set of numerical data can be used for interpolation at any position of the independent variable lying within its two extreme values. The approach of interpolation, described here, can be suitably applied in inverse interpolation also. Newton's forward interpolation formula is valid for estimating the value of the dependent variable under the following two conditions [2]: 1. The given values of the independent variable are at equal interval. 2. The value of the independent variable corresponding

to which the value of the dependent variable is to be estimated lies in the first half of the series of the given values of the independent variable.^[2]

Therefore, the formula derived here is valid for representing a set of numerical data on a pair of variables by a polynomial under these two conditions only. Consequently, there is necessity of searching for some formula for representing a set of numerical data on a pair of variables by a polynomial if the value of the independent variable corresponding to which the value of the dependent variable is to be estimated lies in the last half of the series of the given values, which are at equal interval, of the independent variable. Moreover, there is also necessity of searching for some formula for representing a set of numerical data on a pair of variables by a polynomial if the given values of the independent variable are not at equal interval.

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