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Symmetric 1- Designs from Maximal Subgroup of Degree 1771 Related to Mathieu Group M_{24}

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Abstract- We construct symmetric 1-designs from the primitive permutation representations of degree 1771. We note that the binary row span of the incidence matrices of each design D_k yield the code denoted C_k. We examine the properties of some of the codes C_k where computations are possible.
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I. INTRODUCTION

Suppose G is the Mathieu group M_{24}. The group G acts on the sextet to generates the point stabilizer $2^6 .3. S_6$. The group G acts on this point stabilizer to form orbits. The orbits of this point stabilizers are 1, 30, 280 and 448. For a primitive group G acting on a Ω, it follows from theorem 3.4.1 and 3.4.2 that if we form orbits of the point stabilizer and take their images under the action of the full group represents the blocks of a symmetric 1 - design.

II. SYMMETRIC 1- DESIGN

In this section we examine all designs invariant under G. Table 1 shows Designs from primitive groups of degree 1771 .t Column one represents the 1-design D_k of orbit length k, column two gives the orbit length, column three shows the parameters of the 1-designs D_k and column four gives the automorphism group of the design.

Table 1: Designs from Primitive Group of Degree 1771

Design	orbit length	parameters	Automorphism Group
D90	30	1-(1771,90,90)	M_{24}
D240	280	1-(1771,240,240)	M_{24}
D1440	1440	1-(1771,1440,1440)	A24
D91	31	1-(1771,91,91)	M_{24}
D241	241	1-(1771,241,241)	M_{24}
D1441	1441	1-(1771,1441,1441)	M_{24}
D330	330	1-(1771,330,330)	M_{24}
D1530	1530	1-(1771,1530,1530)	M_{24}
D1680	1680	1-(1771,1680,1680)	M_{24}
D331	331	1-(1771,331,331)	M_{24}
D1531	1531	1-(1771,1531,1531)	M_{24}
D1681	1681	1-(1771,1681,1681)	M_{24}
D1770	1770	1-(1771,1770,1770)	M_{24}

- Proposition 2.1. Let G be the Mathieu simple group M_{24}, and Ω the primitive G-set of size 1771 defined by the action on the cosets of M_{22}:2. Let β = $\{M^g: g \in G\}$ and $D_k = (\Omega, \beta)$. Then the Aut (D_k) is isomorphic to M_{24}
- Proof The only composition factor of Aut(D_k) is M_{24}. This implies that Aut(D_k) is isomorphic M_{24}

III. SOME LINEAR BINARY CODES

We note that the binary row span of the incidence matrices of each design D_k yield the code denoted C_k. We examine the properties of some of the codes C_k where computations are possible.

- Proposition 3.1. Let G be the primitive group of degree 1771 of M_{24} and C a linear code admitting G as an automorphism group. Then the following holds:

i. C_{448} is a, self- orthogonal and doubly even projective [1771, 22,264] binary code. The dual code $C^{\perp}_{448}$ is a [1771, 737, 3] binary code of weight 3.

ii. C_{311} is a projective [1771, 23,264] binary code with 1288 words of weight 264. $C^{\perp}_{311}$ of C_{311} is a [1771, 736, 4] binary code.

- Proof

i. The weight distribution of this code is $C_{448} = 1+ 1288x^{264} + 26565x^{320} + 276828x^{352} +510048x^{360} + 680064x^{376} + 1772771x^{384} + 807576x^{392} + 97152x^{408} + 21252x^{416} + 759x^{448}$.From the weight distribution of $C_{448,}$ we observe that codewords have weights divisible by 4. C_{448} is doubly even. Hence C_{448} is self-orthogonal. The minimum weight of $C^{\perp}_{448}$ code is 3. Hence C_{448} is projective.

ii. The weight distribution of this code is $C_{311} = 1+ 1288x^{264} + 759x^{311} + 26565x^{350} + \ldots$ The minimum weight of $C^{\perp}_{311}$ code is 4. Hence C_{311} is projective.

CONCLUSION

Let *G* be the primitive group of degree 1771 of M_{24} and *C* a linear code and *D* a primitive design admitting *G* as an automorphism group. Then the following holds:

a) There exists a self-orthogonal doubly even projective code.

b) There exist a set of Primitive Symmetric 1-Designs related to M_{24}.

c) Aut((D_k) ~= M_{24}

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