On Class (N+K, MBQ) Operators

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***Abstract-* In this paper, we introduce the class of (n+k, mBQ) operators acting on a complex Hilbert space H. An operator if T ∈B(H) is said to belong to class (n+k, mBQ) if $T^{*2m}T^{2(n+k)}$ commutes with $(T^{*m}T^{n+k})^2$equivalently $[T^{*2m}T^{2(n+k)}, (T^{*m}T^{n+k})^2] = 0$, for a positive integers n and m. We investigate algebraic properties that this class enjoys. have. We analyze the relation of this class to (n+k, m)-powerclass (Q) operators.**

***Indexed Terms-* (n,m)-power Class (Q),Normal ,Binormal operators , n-power class (Q), (BQ) operators , (n+k,mBQ) operators.**

I. INTRODUCTION

H denotes Hilbert space over the complex field throughout this paper while B(H) the Banach algebra of all bounded linear algebra on an infinite dimensional separable Hilbert space H. A bounded linear operator T is said to be in class (Q) if $T^{*2}T^{2} = (T^{*}T)^2$ (2), (n,m)-power class (Q) if $T^{*2m}T^{2n} = (T^{*m}T^{n})^2$for positive integers n and m (1). The class of (Q) operators was expanded to many classes such as the following classes, almost class (Q) (4), n-power class (Q) (2), (α, β)-class (Q) (3) , K* Quasi-n- Class (Q) Operators (6) and quasi M class (Q) . An operator T ∈B(H) is said to belong to class (BQ) if $T^{*2}T^{2}(T^{*}T)^2 = (T^{*}T)^2T^{*2}T^{2}$ (5), T ∈B(H) is said to belong to class (n+k, mBQ) if $T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$. A conjugation on aHilbert space H is an anti-linear operator C from Hilbert space H onto itself that satisfies Cξ, Cζi= hζ, ξifor every ξ, ζ ∈H and $C^2 = I$. An operator T is said to be complex symmetric if $T = CT^{*}C$.

II. MAIN RESULTS

Theorem 1. Let T ∈ B(H) be such that T ∈ (n+k, mBQ), then the following holds for (n+k, mBQ);

i. λT for any real λ
ii. Any S ∈ B(H) that is unitarily equivalent to T.
iii. The restriction T M to any closed subspace M of H.

Proof. i. The proof is straight forward. ii. Let S ∈B(H) be unitarily equivalent to T, then there exists a unitary operator U
∈B(H) with

$S^{n+k} = U^{*}T^{n+k}U$and $S^{*m}= U^{*}T^{*m}U$ for non-negative integers n and m. Since T ∈ (n+k, mBQ), we have;
$T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$, hence

$S^{*2m}S^{2(n+k)}(S^{*m}S^{n+k})^2 = UT^{*2m}U^{*}UT^{2(n+k)}U^{*}(UT^{*m}U^{*}UT^{n+k}U^{*})^2$
$= UT^{*2m}U^{*}U^{*}T^{2(n+k)}U^{*}UT^{*m}U^{*}UT^{*m}U^{*}UT^{n+k}U^{*}UT^{n+k}U^{*}$
$= UT^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2U^{*}$
$= U(T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}U^{*}$
and
$(S^{*m}S^{n+k})^2S^{*2m}S^{2(n+k)} = (UT^{*m}U^{*}UT^{n+k}U^{*})^2UT^{*2m}U^{*}UT^{2(n+k)}U^{*}$
$= UT^{*m}U^{*}UT^{n+k}U^{*}UT^{*m}U^{*}UT^{n+k}U^{*}UT^{*2m}U^{*}UT^{2(n+k)}U^{*}$
$= UT^{*m}T^{n+k}T^{*m}T^{n+k}T^{*2m}T^{*2m}U^{*}$
$=U(T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}U^{*}$
Thus, S is unitarily equivalent to T.

iv. If T is in class (n, mBQ), then;
$T^{*2m}T^{2(n+k)}(T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2T^{*2m}T^{2(n+k)}$.
Hence;
$(T/M)^{*2m}(T/M)^{2(n+k)}\{(T/M)^{*m}(T/M)^{n+k}\}^2$
$=(T/M)^{*2m}(T/M)^{2(n+k)}\{(T/M)^{*m}(T/M)^{n+k}\}^2$
$= (T^{*2m}/M)(T^{2(n+k)}/M)\{(T^{*m}/M)(T^{n+k}/M)\}\{(T^{*m}/M)(T^{n+k}/M)\}$
$= \{(T^{*m}T^{n+k})^2/M\}\{T^{*2m}T^{2(n+k)}/M\}$
$= \{(T^{*m}/M)(T^{n+k}/M)\}^2(T/M)^{*2m}(T/M)^{2(n+k)}$

Thus T/M ∈ (n+k, mBQ).

- Theorem 2. If T ∈ B(H) is in (n+k,m)-power Class (Q), then T ∈ (n+k,mBQ).

Proof. If $T \in (n+k)$-power (Q), then
$T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$
post multiplying both sides by $T^{*2m}T^{2(n+k)}$;
$T^{*2m}T^{2(n+k)}\, T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$
$T^{*2m}T^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k} = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$
$T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$.

Theorem 3. Let $S \in (n+k, mBQ)$ and $T \in (n+k, mBQ)$. If both S and T are doubly commuting, then ST is in $(n+k, mBQ)$.

Proof.
$(ST)^{*2m}(ST)^{2(n+k)} ((ST)^{*m}(ST)^{n+k})^2$
$=S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)} ((ST)^{*m}(ST)^{n+k}) ((ST)^{*m}(ST)^{n+k}) = S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)} ((S^{*m}T^{*m}) (S^{n+k}T^{n+k})) ((S^{*m}T^{*m}) (S^{n+k}T^{n+k}))$
$=S^{*2m}T^{*2m}S^{2(n+k)}T^{2(n+k)}S^{*m}T^{*m}S^{n+k}T^{n+k}S^{*m}T^{*m}S^{n+k}T^{n+k}S^{*m}T^{*m}S^{n+k}T^{n+k}$
$=T^{*2m}T^{2(n+k)}S^{*2m}S^{2(n+k)}S^{*m}S^{n+k}S^{*m}S^{n+k}T^{*m}T^{n+k}T^{*m}T^{n+k}$
$=T^{*2m}T^{2(n+k)}\, S^{*2m}S^{2(n+k)}\, (S^{*m}S^{n+k})^2 T^{*m}T^{n+k}T^{*m}T^{n+k}$
$=T^{*2m}T^{2(n+k)}\, (S^{*m}S^{n+k})^2 S^{*2m}S^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k}$ (Since $S \in (n+k, mBQ)$).
$= (S^{*m}S^{n+k})^2 T^{*2m}T^{2(n+k)}\, T^{*m}T^{n+k}T^{*m}T^{n+k}S^{*2m}S^{2(n+k)}$
$=(S^{*m}S^{n+k})^2 T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 S^{*2m}S^{2(n+k)}$
$=(S^{*m}S^{n+k})^2 (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}\, S^{*2m}S^{2(n+k)}$ (Since $T \in (n+k, mBQ)$).
$=((S^{*m}S^{n+k}) (T^{*m}T^{n+k}))^2 T^{*2m}S^{*2m}T^{2(n+k)}\, S^{2m}$
$=((S^{*m}T^{*m}) (S^{n+k}T^{n+k}))^2 S^{*2m}T^{*2m}S^{2(n+k)}\, T^{2(n+k)}$
$= ((ST)^{*m}(ST)^{n+k})^2 (ST)^{*2m}(ST)^{2(n+k)}$

Thus $ST \in (n+k, mBQ)$.

- Theorem 4. Let $T \in B(H)$ be a class $(n+k, mBQ)$ operator such that $T = CT^{*}C$ for positive integers n and m with C being aconjugation on H. If C is such that it commutes with $T^{*2m}T^{2(n+k)}$ and $(T^{*m}T^{n+k})^2$, then T is an $(n+k,m)$-power class (Q) operator.

Proof. Let $T \in (n+k, mBQ)$ and complex symmetric, then we have; $T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$
and $T = CT^{*}C$.

hence;
$T^{*2m}T^{2(n+k)} (T^{*m}T^{n+k})^2 = (T^{*m}T^{n+k})^2 T^{*2m}T^{2(n+k)}$
$T^{*2m}T^{2(n+k)}CT^{n+k}CCT^{*m}CCT^{n+k}CCT^{*m}C$
$=(T^{*m}T^{n+k})^2 CT^{n+k}CCT^{*m}CCT^{n+k}CCT^{*m}C$.
$T^{*2m}T^{2(n+k)}CT^{n+k}T^{*m}T^{n+k}T^{*m}C$
$=(T^{*m}T^{n+k})^2 CT^{n+k}T^{*m}T^{n+k}T^{*m}C$
$T^{*2m}T^{2(n+k)}CT^{2(n+k)}T^{*2m}C = (T^{*m}T^{n+k})^2 CT^{*m}T^{n+k}T^{*m}T^{n+k}C$
$T^{*2m}T^{2(n+k)}\, CT^{*2m}T^{2(n+k)}\, C = (T^{*m}T^{n+k})^2 C (T^{*m}T^{n+k})^2 C$.
C commutes with $T^{*2m}T^{2(n+k)}$ and $(T^{*m}T^{n+k})^2$ hence we obtain;
$T^{*2m}T^{2(n+k)}\, T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2 (T^{*m}T^{n+k})^2$.
which implies;
$T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$ and thus $T \in (n+k,m)$-power class(Q).

Theorem 5. Let $T \in B(H)$ be $(n+k-1, m)$-class (Q) operator, if T is a complex symmetric operator such that C commutes with $(T^{*m}T)^2$ for a positive integerm, then T is an $(n+k, m)$-power class (Q) operator.

Proof. With T being complex symmetric and $(n-1, m)$-class (Q), we have;
$T = CT^{*}C$ and $T^{*2m}T^{2(n+k-1)} = (T^{*m}T^{n+k-1})^2$.
We obtain;
$T^{*2m}T^{2(n+k-1)}\, T^{2} = (T^{*m}T^{n+k-1})^2 T^{2}$.
hence;
$T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k-1})^2 T^{2}$.
$T^{*2m}T^{2(n+k)} = T^{*2m}T^{2(n+k-1)}\, T^{2} = T^{2(n+k-1)}\, T^{*2m}T^{2}$
$T^{*2m}T^{2(n+k)} = T^{2(n+k-1)}\, T^{*m}T^{*m}TT=$
$T^{2(n+k-1)}\, CTCCTCCT^{*m}CCT^{*m}C = T^{2(n+k-1)}\, CTTT^{*m}T^{*m}C$.
$=T^{*2m}T^{2(n+k)} = T^{2(n+k-1)}\, CT^{2}T^{*2m}C = T^{2(n+k-1)}\, C (T^{*m}T)^2 C$

Since C commutes with $(T^{*m}T)^2$ we obtain; $T^{*2m}T^{2(n+k)} = T^{2(n+k-1)} (T^{*m}T)^2 CC = T^{2(n+k-1)}\, T^{*2m}T^{2}CC = T^{2(n+k-1)}\, T^{2}T^{*2m}CC = T^{*2m}T^{2(n+k)} = (T^{*m}T^{n+k})^2$

Hence T is n+k-power class (Q).

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