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A Study on Extension of Double Acceptance Sampling Plans Based on Truncated Life Tests on The Inverse Rayleigh Distribution

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Abstract- Sequential probability ratio test analysis (SPRT) which was the special case for multiple acceptance plans was develop using the inverse Rayleigh predicting software for the life time random variable for a truncated life test. In this paper, double acceptance sampling extends by Zoramawa et al (2018) has been extended to sequential sampling plans in terms of obtaining the minimum number of sample sizes necessary to obtained specified average life time under a given consumer's risk. The lower proportion, higher proportion and the coefficient function where obtained, for different values of β_i ***and*** α_i ***and show the graphical performance of the model either to reject, to accept, to continue or terminate the procedure.***

I. INTRODUCTION

1.0 Background of the Study

The Acceptance Sampling is a practice whereby a sample is tested from a population (lot), and a decision to accept or reject that entire population (lot) is based on the test results of the sample. Acceptance Sampling originated in the 1930's at Bell Labs through the work of Dodge (1943) and was later popularized during World War II by the U.S. Military for munitions (bullets) production. During the war, many bullets were produced and there was no economic way to test them all. Additionally, some of the testing was destructive, rendering the munitions unusable, so 100% inspection was impossible. Acceptance Sampling became the compromise between no inspection and 100% inspection, and allowed the manufacturers to infer the overall "quality" of an entire lot while only testing a fraction of the entire lot. Over time, acceptance sampling has also become advantageous for other companies who have faced destructive inspection, or when the cost associated with 100% inspection was not economical, or where the risk of passing along a defect is low Rosaiah, *et al* (2006).

Acceptance sampling plans is an inspection procedures and decision making used to determine whether to accept or reject a lot this involves both the producer (supplier) of product and the consumers (buyers). Consumers need acceptance sampling to limit the risk for rejecting a good quality material or accepting bad quality product. Consequently, the consumers, sometimes in conjunction with the producers through contractual agreements, specified the parameter of the plan, any company can be both a producer of product purchased by another company and a consumer of a products or raw material supplied by another Montgomery, (2013)

Acceptance sampling plans is a quality control method used to accept or reject a lot testing in inspecting of a product, the purpose of acceptance sampling is to make a determination about a product, accept the lot or reject it rather than to estimate the quality of the entire product Teh,*et al* (2016).

The development of double sampling plan is the most popular plan because of its simplicity and ease of administration. In the double sampling plan, first, aims at reducing the average number of observations needed to yield a decision; sample is taken from the lot, and if the number of defectives is greater than a specified criterion, then the submitted lot is rejected. If the number of defectives is smaller than another criterion, the submitted lot is accepted, or else another sample is taken, and the final decision is made based on the results of both samples Zoramawa *et al* (2018).The double acceptance sampling is used to minimize the risk of a items because it brings another opportunity for acceptances in case of the rejection initially occur in single plans, the consumers risk which is the probability of accepting a bad items (lots) when in the actual sense is supposed to be accepted Zoramawa, *et al* (2018).

The SPRT was initially developed by Wald (1947) for quality control problems during World War II. It has many extensions and applications: such as in clinical trial and in quality control. The original development of the SPRT is used as a statistical device to decide which of two simple hypotheses is more correct. In Wald's SPRT, if certain conditions are met during the data collection decisions are taken with regard to continuing the data collection and the interpretation of the gathered data. Wald's procedure is particularly relevant if the data is collected sequentially. Sequential Analysis is different from Classical Hypothesis Testing were the number of cases tested or collected is fixed at the beginning of the experiment. In Classical Hypothesis Testing the data collection is executed without analysis and Consideration of the data. After all data is collected the analysis is done and conclusions are drawn. However, in Sequential Analysis every case is analyzed directly after being collected, the data collected up to that moment is then compared with certain threshold values, incorporating the new information obtained from the freshly collected case. This approach allows one to draw conclusions during the data collection, and a final conclusion can possibly be reached at a much earlier stage as is the case in Classical Hypothesis Testing. Prasad, Ramadevi and Sridevi, (2015)

First idea of a sequential sampling plan test procedure goes back to Dodge (1943) who constructed a double sampling procedure. According to this scheme, the decision whether or not a second sample should be drawn depends on the observations of the first sample. This method was expanded to multiple sampling by Walter (1943) for the case of testing the mean of a binomial distribution. Dodge and Torrey (1951) presented these schemes in recognition of the fact that they require, on the average, a smaller number of observations than single sampling Wald, (1947). In particular they stated that a sequential test procedure might be constructed that would control error to the same extent as the best current procedure based on a predetermined number of trials. Inverse Rayleigh Distribution was first derived by was originally derived by Rayleigh (1880) in the connection with problems in the field of acoustics; is a continue probability distribution for non-negative random variables which naturally arises and received more attention by the researcher due to its application in a different field of magnetic resonance imaging, field of nutrition, survival and hazard rate functions Physical Ocean graphic.

The probability density function (PDF) written as

$$f\left(x,\delta\right)=\frac{x}{\delta^2}e^{-\frac{x^2}{(2\delta)^2}},x\geq 0 \qquad 1.1$$

Where δ is the scale parameter of the distribution the cumulative distribution function for Rayleigh is given as

$$f\left(x,\delta\right)=1-e^{-}\frac{x^2}{\left(2\delta^2\right)} \text{ for } x \in (0,\delta) \qquad 1.2$$

The mean of the Rayleigh random variable is given as

$$u(x)=\delta\sqrt{\frac{\pi}{2}}=1253\delta \qquad 1.3$$

1.1 Statement of the Problem

Sequential probability ratio test analysis treats the sample size for a particular procedure and aims to make it as small as possible and still make a decision. The ability to potentially reduce the sample size required to make a decision in an experiment has numerous applications because it leads to the conserving of resources, making funding easier to appropriate, Sequential probability ratio test was an extension of single and double sampling plan that determine the minimum number of observations required to terminate a sample Opperman and Ning, (2019). Rosaiah, and kantam (2005) developed acceptance sampling plan procedure for inverse Rayleigh distributions mean under a truncated life, the work try to address the problem of producers‘s risk by determining the minimum number of sample size necessary to give assurances of accepting the lot but it only uses one sample size, Zoramawa, *et al* (2018) extended the single acceptance sampling plans (SASP) to double acceptance sampling (DASP) and the result requires minimum number of observations to terminate the inspection by finding n_1 and n_2 which is necessary to assert that the life of the product meet the required specification. However, in a double sampling plan, after the first sample is tested, there are three possibilities accept the lot, reject the lot, and take a second sample, and the procedure is to combine the results of both samples and make a final decision based on that information, there is risk in accepting bad lot and rejecting good lot and also costly time consuming. We proposed the Sequential probability ratio test analysis (SPRT) which was the special case for multiple acceptance plans. Under sequential sampling, sample are taken one at time, until a decision is made on the lot or process sampled, after each item is taken, a decision is made to accept, reject or continue sampling.

1.2 Aim and Objectives

The aim of this research was to extend the double acceptance sampling plan to sequential probability ratio test (SPRT) which is ultimate extension of multiple sampling plans to run an inverse Rayleigh distribution for Predicting Software Reliability lots with the following objectives:

1. To compute the average sample number (ASN) using the SPRT parameters
2. To determine the probability of acceptance using the sequential probability ratio test (SPRT)
3. To plot the required OC curve outlier's regions for the conduct sampling plan.

1.3 Significance of the Study

This research is significant in the sense that sequential probability ratio test (SPRT) is an extension of the double acceptance sampling plans for hypothesis testing. The idea of sequential probability ratio test (SPRT) constructed as a sequential test of one simple hypothesis against another is to sample the batch of a lots sequentially until a decision can be made whether the batch of the product is conforms to specification and can be accepted or that is should be rejected on continue sampling, which we shall use truncated life tests on the inverse Rayleigh distribution.

1.4 Scope and Limitation of the Study

This research is limited to early work of Zoramawa, *et al* (2018). The scope is to extended the double acceptance sampling plans of Zoramawa, *et al* (2018) to sequential probability ratio test (SPRT) using the inverse Rayleigh distribution for predicting software reliability.

1.5 Research Hypothesis

The adopt confidence limit used by Rosaiah, et *al* (2005), Zoramawa, *et al* (2018) respectively, we intend to extended the single and double acceptance sampling plans to sequential probability ratio test (SPRT) using the hypothesis testing for null hypothesis H_O against alternative hypothesis H_1

1.6 Predicting Software Reliability

Critical business applications requires reliable software but developing reliable software is one of the most difficult problems facing the software industry, therefore experiment with software reliability growth models show that simple models of execution time and cumulative defects that is close to the number report in the field Wood, (1996) software reliability $R(x/t)$ is define as the probability that a software does not occur in the time interval $\left(t, t+r\right)$ given that the last failure occurred at testing time $t\left(t \geq 0, x > 0\right)$. That is $R\left(x+t\right) = e^{-}\left[m(t+x) - m(t)\right]$

Where $m(t)$ is the expected cumulative number of failures, which known as the mean value function, for special case when $t = 0$

Then $R(x/0) = e^{-m(x)}$ and when $t = \infty$ then $R(x/\infty) = 1$

1.7 GLOSSARY OF SYMBOLS

r: Number of testers
n: Sample size
N: Population size
d: Defectives items
i: Allowable acceptance number
p: Quality level
α: Producer's risk (probability of rejecting a good lot)
β: Consumer's risk (probability of accepting a bad lot)
p_a (p): Probability of lot acceptance
t_o: Test termination
$\frac{\mu}{\mu_0}$: Mean ratio

$m(t)$: Expected number of software failures by time

a: Total number of software errors to be eventually detected

b: Exponential index

$R(t)$: Reliability function of software by time T for a mission

II. LITERATURE REVIEW

2.0 Introduction

Sequential analysis starts with testing a simple null hypothesis against a simple alternative hypothesis. The fixed sample size problem of this classic test was solved by Neyman and Pearson (1933) who laid the theoretical foundation of likelihood-based hypothesis testing. The sequential probability ratio test (SPRT), formulated via the boundary crossing of the likelihood ratio statistic, is proved to be optimal in terms of minimal expected sample size for fixed type I and type II error probabilities Wald and Wolfowitz, (1948). Due to the usefulness of the sequential probability ratio test in development work on military and naval equipment, it was classified Restricted within the realm of the Espionage Act. Found the operating characteristic curve (OC) of the Sequential probability ratio test SPRT for the case of the binomial distribution. Wald then created a formula to find the average sample number ASN for the sequential probability ratio test SPRT.

2.1 Review of Related Literature

Rosaiah, *et al* (2008) developed acceptance sampling plan for half logistic distribution, Rosaiah and Kantam (2005) for logistic distribution, Baklizi, (2003), Ghosh and Ramamoorthi (2003) propose one way of handling composite hypothesis by SPRT to integrate out nuisance parameter under both the null and alternative hypothesis.

Rosaiah and Kantam (2005) determine the acceptance sampling based on the inverse Rayleigh distribution for economic quality control. Rosaiah, *et al* (2006) developed the reliability plans for exponential log logistic distribution and Balakrishnan, (2007) for generalized Birnbaum-Sanders distribution. For the generalized exponential distribution, Shi,*et al* (2007) change of measure of techniques for the computation of small probabilities have been employed under various setting, Rosaiah, Kantam, Prasad and Reddy (2008) for inverse Rayleigh distribution, Aslam, (2007) double acceptance sampling based on truncated life test for inverse Rayleigh distribution. Darkhovsky, (2009) group acceptance sampling plans for truncated life tests based on inverse Rayleigh distribution, and logistics distribution,

Aryal and Tsokos (2011) proposed the transmuted weibull and studied the various structural properties of the model for analyzing reliability data, distribution, Russell,*et al* (2012) propose two approaches for problem solution, one based on frequencies method and the other on a Bayesian method, Muhammad, *et al* (2013) discussed the repetitive group sampling plans utilized to find out the number of group acceptance sampling, Sudamani and Sutharani (2013) double acceptance sampling plans based on truncated life test in Rayleigh distribution using minimum angle method for the Pareto distribution of the second kind, Cox, (2013) mention several applications such as the one hit and two hit models of binary dose response and testing the interactions in a balanced of factorial experiment.
Bacanli and Icen (2013) in their paper uses a well-known test procedure of statistical science called as Sequential Probability Ratio Test (SPRT) is adopted for inverse Rayleigh distribution model in assessing the reliability of developed software. In their research, SPRT requires considerably less number of observations when compared with the other existing testing procedures. Hence Sequential Analysis of Statistical Science could be adopted to decide upon the reliable / unreliable of the developed software very quickly. The paper proposes the performance of SPRT on inverse Rayleigh distribution model and analyzed the results by applying on data sets. The Maximum Likelihood Estimation is used for estimation of parameters.

Li, Liu and Xu (2016) consider the problem of testing two separate families of hypotheses via a generalization of the sequential probability ratio test. In particular, the generalized likelihood ratio statistic is considered and the stopping rule is the first boundary crossing of the generalized likelihood ratio statistic. Which show that this sequential test is asymptotically optimal in the sense that it achieves asymptotically the shortest expected sample size as the maximal type I and type II error probabilities tend to zero. Yahya, (2007) proposes a robust fault detection method with an Artificial Neural Network-Multi-Layer Perceptron (ANN-MLP) and a statistical module based on Wald's sequential probability ratio test (SPRT). To detect a fault, this method uses the mean and the standard deviation of the residual noise obtained from applying a NARX (Nonlinear Auto-Regressive with exogenous input) model. To develop the neural network model, the required training and testing data were generated at different operating conditions. To show the effectiveness of the proposed fault detection method, it was tested on a realistic fault of a distillation plant at the laboratory scale. Nakamura, *et al* (2016) proposed sequential testing procedure to determine the minimum dose with a threshold effect. Gardonyi, *et al* (2019) uses a novel method called Scaled Sequential Probability Ratio Test (SSPRT) produces 2D array of data via special cumulative sum calculation. A peak determination algorithm has also been developed to find significant peaks and to store the corresponding data for further evaluation. The method provides straight information about the endpoints and possible duration of the detected events as well as shows their significance level. The new method also gives representative visual information about the structure of detected events.

Sukhdev, *et al* (2019) addresses the problem of double and group acceptance sampling plans for an inverse weibull distribution based on truncated life test. Singh, *et al* (2019) discussed a comparison between single and double acceptance sampling plans based on inverse weibull distribution, Chen, Li, *et al* (2019) considered the prediction of duration and final income success and failure for solving complex problem during task completion process by making use of process data recorded in the computer files.

Steland (2015) derive new acceptance sampling plans that control the overall operating characteristics, the acceptance sampling in particular a modified sampling and the case on the accuracy for spatial batch sampling on the accuracy of the estimation. Ewart and Thomas (1975) have analyzed a random walk model for two choice reaction item on the assumption that the two probability density functions (PDFs) of the step – size each PDF and demonstrated the critical role played by the symmetry of the moment generating functions (MGF) of the step size in the determination of whether or not the error and correct reaction times are equal. Also, Stute (1996) investigate the properties of the sequential probability ratio test when data are at risk of being censored; it turns out that the stopping boundaries are the same for completely observable data. But the average sample size increase as censoring becomes more substantial. Lens and Wilrich (2006) propose a simple and easy to design, special case of sequential sampling plans by attribute, name cseq-1 sampling plans having acceptance numbers not greater than one, and analyzed the properties of these plans compare them to the properties of the widely used of sampling procedures. Davida, Amsden and Butler (1991) considered the sampling plans to be a statistical process control techniques like the control chart discussed in the other modules, which give the sampling plans tools the quality to be used in conjunction with SPC tools and quality Tapiero (1996) explain the purpose of acceptance simply which is to provide for associated with accepting a lot are within specified limit. It is necessary to specify the risk, state clearly how sample data abroad managerial approach to inspection and acceptance sampling plans.

Aslam and Ali (2019) propose the acceptance sampling plans as an important field of statistical quality control (SQC) to inspect the final product before it can be realized for consumer's use, the testing of items including computers, mobile phones, and automobiles need the acceptance sampling plans schemes to solve the life testing problems. Other procedures and the consumers need efficient acceptance sampling plans schemes. Singh, *et al* (2019) considered repetitive acceptance sampling for truncated life test in which the life time of the product follows the generalized Pareto distribution in which the plan requires less sample size than the acceptance sampling plans. Zoramawa, *et al* (2018) comes up with a procedure for computing double acceptance sampling based on truncated life tests on inverse Rayleigh distribution operation characteristic curve and average sample number (ASN) which was best fit than the single acceptance sampling plan in which the decision of the first and second is combined in order to reach a decision whether to accept or reject the lot.
However, from the above literature the researchers have address the acceptance sampling based on life truncated tests on single and double acceptances sampling plans, Rosaiah and Kantam (2005),

Sukhdev,*et al* (2019), Sudamani and Sutharani (2013), Muhammad,*et al* (2012), Darkhovsky, (2009), Zoramawa, *et al* (2018). In this research we considered the special case of lengthening the double acceptance sampling to sequential probability ratio test SPRT in other to make decision either to reject, accept or continue sampling in the sense not only to obtained "n" the required number and terminate the sample plans. We therefore proposed Sequential probability ratio test (SPRT) which units are selected from the lot one at a time, and following inspection of each unit, a decision is made to accept the lot, reject the lot or select another unit after each item is taken.

III. MATERIAL METHODOLOGY

3.0 THE SEQUENTIAL PROBABILITY RATIO TEST (SPRT) COMPUTATIONS

Sequential probability ratio test SPRT was originally developed as an inspection tool to determine whether a given lot meets the production requirements. Basically, a sequential is the most discriminating acceptance sampling procedure involves in making a decision as to disposition of the lot or resample successively test is a method by which items are tested in sequence one after another, these methods may be regards as multiple sampling plans with sample size one and no upper limit on the number of samples to be taken. Sequential approach provides essentially optimum efficiency in sampling that is an average sample number (ASN) as low as possible.

Sometimes acceptance is not allowed at the early stages of multiple samples; however, the rejection can occur at any stage.

The equivalent hypothesis is given by the following two tests of significance are applied to the data accumulated.

$$H_0: p = p_1$$
$$H_1: p = p_2$$

H_O: the lot is of acceptable quality level (AQL, p_1)
H_1: The lot is of reject able quality level RQL (LTDP, p_2)
α = p (rejected H_O/H_O is true)
β = p (accept H_O/H_O is false)

When designing an item-by-item sequential sampling plan, four parameters of the AQL, the producer's risk α (the probability of rejecting a lot with AQL quality), LTDP and the consumer's risk β (the probability of accepting a lot with LTDP quality) must be determined prior to determining the acceptance and rejection line. Both the acceptance and rejection number must be integer, the acceptance number is the next integer less or equal to, Y_1 and the rejection number is the next integer greater than or equal to Y_2 .

Consider the ratios

$$R(n, y) = \frac{f(y,p_1)}{f(y,p_2)} \qquad 3.1$$

Where $f(y, p)$ is the probability function which can be Poisson, or Hypergeometric or binomial?

Assign formulas for the construction and evaluation of sequential plans values of $p_{1,} p_{2,} \alpha$ and β have derived by Wald (1947) and Statistical Research Group (1945) which are as follows:

$$h_1 = \frac{\log\left(\frac{1-\alpha}{\beta}\right)}{\log\left(\frac{p_2}{p_1}\right) + \log\left(\frac{1-p_1}{1-p_2}\right)} \qquad 3.2$$

$$h_2 = \frac{\log\left(\frac{1-\beta}{\alpha}\right)}{\log\left(\frac{p_2}{p_1}\right) + \log\left(\frac{1-p_1}{1-p_2}\right)} \qquad 3.3$$

$$s = \frac{\log\left(\frac{1-p_1}{1-p_2}\right)}{\log\left(\frac{p_2}{p_1}\right) + \log\left(\frac{1-p_1}{1-p_2}\right)} \qquad 3.4$$

either common or natural logarithms can be used in their computations provided they are consistent, then the acceptance and rejection line are determined as

$Y_1 = -h_1 + sn$ (Acceptance line), $Y_2 = h_2 + sn$ (Rejection line)

These formulas are sometimes expressed as

$$h_1 = \frac{b}{g_1 + g_2} \quad 3.5$$

$$h_2 = \frac{a}{g_1 + g_2} \quad 3.6$$

$$s = \frac{g_2}{g_1 + g_2} = \frac{g_2}{G} \quad 3.7$$

Where

$$a = \log \frac{1-\beta}{\alpha} \quad 3.8$$

$$b = \log \frac{1-\alpha}{\beta} \quad 3.9$$

$$g_1 = \frac{p_2}{p_1} \quad 3.10$$

$$g_2 = \frac{1-p_1}{1-p_2} \quad 3.11$$

$$G = g_1 + g_2 \quad 3.12$$

Regions

Acceptance region:
Accept if $R(n, y) \leq B$
i.e., if $y_1 \leq -h_1 + sn$ 3.13

Rejection region:
Reject if $R(n, y) \geq A$
i.e., if $y_2 \geq h_2 + sn$ 3.14

Continue sampling: Continue if $[B \leq R(n, x) \leq A]$
i.e., if $-h_1 + s_n < x < h_1 + sn$ 3.15

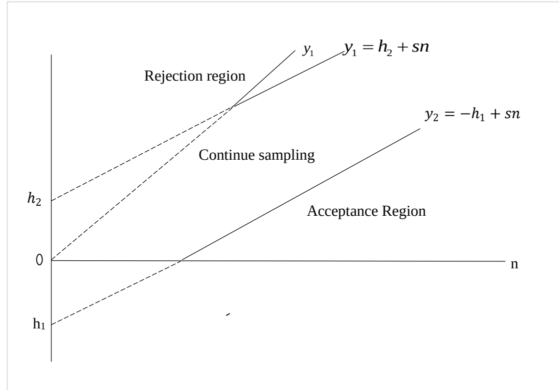


Fig. 1 Graphical Performance of Sequential Sampling Plan

$y_2 = -h_1 + sn$ (Acceptance line), $y_1 = h_2 + sn$ (Rejection line)

Vertical axis is the total number of observed non-conforming items, then the operation procedure is given in the following:

i. If the plotted point falls within the limit lines the process continues by drawing another sample
ii. When the plotted points fall on or above the upper line, the lot is rejected
iii. When the plotted points fall on or below the lower line, the lot is accepted

3.1 AVERAGE SAMPLE NUMBER (ASN)

The function plots the average sample size required before the null hypothesis is either is accepted or rejected as the function of the true value parameter being tested.

The ASN can be plotted from the following fixed points:

$$\text{ASN} = \frac{h_1}{s}, \qquad for p = 0$$

$$\text{ASN} = \frac{(1-\alpha)h_1 - \alpha h_2}{(s - p_0)}, \qquad for p = p_0$$

$$\text{ASN} = \frac{h_1 h_2}{s(1-s)}, \qquad for p = s$$

$$\text{ASN} = \frac{(1-\beta)h_2 - \beta h_1}{p_1 - s}, \qquad for p = p_1$$

$$\text{ASN} = \frac{h_2}{1-s}, \qquad for p = 1$$

The general formula for ASN is given as

$$ASN = \frac{p_a \log\left(\frac{\beta}{1-\alpha}\right) + (1-p_a)\log\left(\frac{1-\beta}{\alpha}\right)}{p \log\left(\frac{p_2}{p_1}\right) + (1-p)\log\left(\frac{1-p_2}{1-p_1}\right)} \quad 3.16$$

3.2 OPERATING CHARACTERISTICS CURVE

The operating characteristics (OC) curve described the probability and discriminatory power of the sampling plan that is shows the probability that a lot submitted with a certain fraction defective will be either accepted, rejected or continue sampling using a sequential probability ratio test analysis. accepting a lot as a function of the lots quality where a lot is a batch or a section is continuing work, with the aid of the curve it is possible to quantify whether the level is reasonable and thus good the acceptance scheme is as a whole, the shape of the curve is dictated by the acceptance constant (k) and the number of the sample (n).

The various values of the parameter being tested, the OC curve can be plotted as

$$p = \frac{1-\left[(1-p_2)/(1-p_1)\right]^h}{\left(\frac{p_2}{p_1}\right)^h - \left[(1-p_2)/(1-p_1)\right]^h} \qquad 3.17$$

$$p_a = \frac{\left(\frac{1-\beta}{\alpha}\right)^h - 1}{\left(\frac{1-\beta}{\alpha}\right)^h - \left(\frac{1-\alpha}{\beta}\right)^h} \qquad 3.18$$

Where $h(\rho)$ is the quantity that depends on the unk nown parameter ρ .

3.3 SEQUENTIAL PROBABILITY RATIO TEST (SPRT)

The sequential probability ratio test SPRT limits for each hypothesis will be compute and plotted using R package, excel and SISA software package by assigning SPRT for null hypothesis H_0: $P=P_1$ with the alternative hypothesis H_1: $P=P_2$

Sample average number ASN and operating characteristics (OC) curve will be computed at P=0 and P=1 and these values will determine whether to accept, reject continue or no decision is made on the ongoing sample, the sequential probability ratio test SPRT plots gives a pair of parallel line for d_1 and d_2 if at any stage n_i values fall above, within or below the lines that will help the experimenter to accept, reject, continue or terminate t he sample.

IV. RESULT AND DISCUSSION

Wood, (1996) in predicting software reliability produces four major releases of defective result Zoramawa, *et al* (2018) consider release 4 and applied double acceptance sampling plans based on truncated life tests for inverse Rayleigh distribution, in this research we are going to considers Rao, (2013) inverse Rayleigh software reliability growth model for dataset 1,2, and 4. However, we applied the sequential probability ratio tests (SPRT) wit β values 0.75, 0.90, 0.95 and 0.99 respectively and α value 0.01 for data set 1, 0.02 for data set 2, 0.03 for data set 4 with constant values of p_1 =0.01 (AQL) and p_2=0.05 (LTPD) from time T (hours).

We therefore draw the following research hypothesis for sequential probability ratio test (SPRT):

Hypothesis: $\beta=\beta_i$ and $\alpha=\alpha_i$

H_0= $p=p_1=0.01$

H_1 =$P=P_2=0.05$

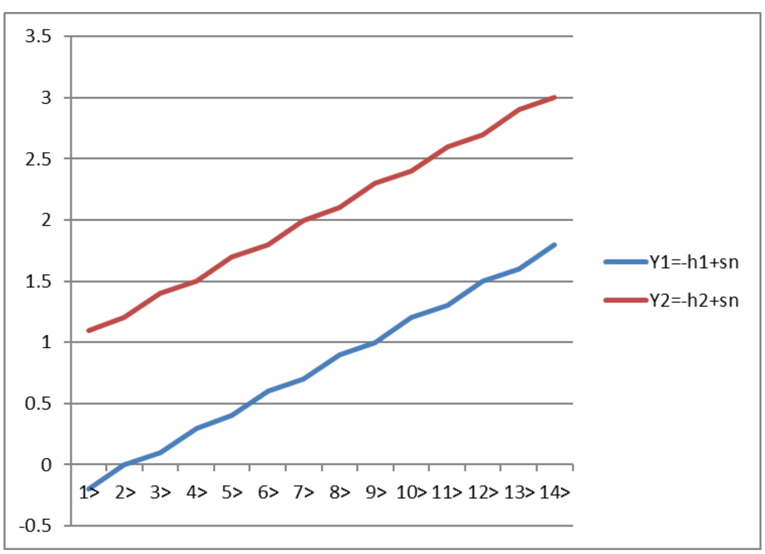
DATE SET 2: t_i (i= 1, 2…,14): $\beta = 0.75, 0.90, 0.95, 0.99$, α = 0.01, $p_1 = 0.01, p_2 = 0.05$

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14
sample	519	968	1430	1893	2490	3058	3625	4422	5218	5823	6539	7083	7487	7846

Sequential probabilities
Lower prop. 0.01
Higher prop. 0.5
Alpha 1%
Power 75%
Experiments 14

General Info				
Regression Coefficient: 0.14866				
Low Intercept: 0.2995				
High Intercept: 0.93958				
SEQUENTIAL PROBABILITY RATIO				
trial	lower limit		higher limit	
1>	-0.2	continue	1.1	Continue
2>	0	continue	1.2	61.80%
3>	0.1	4.90%	1.4	46.20%
4>	0.3	7.40%	1.5	38.40%
5>	0.4	8.90%	1.7	33.70%
6>	0.6	9.90%	1.8	30.50%
7>	0.7	10.60%	2	28.30%
8>	0.9	11.10%	2.1	26.60%
9>	1	11.50%	2.3	25.30%
10>	1.2	11.90%	2.4	24.30%
11>	1.3	12.10%	2.6	23.40%
12>	1.5	12.40%	2.7	22.70%
13>	1.6	12.60%	2.9	22.10%
14>	1.8	12.70%	3	21.60%
After trial 14				
Reject if:				
- Less than 12.7% are positive				
- More than 21.6% are positive				
Otherwise continue				
or accept no difference				

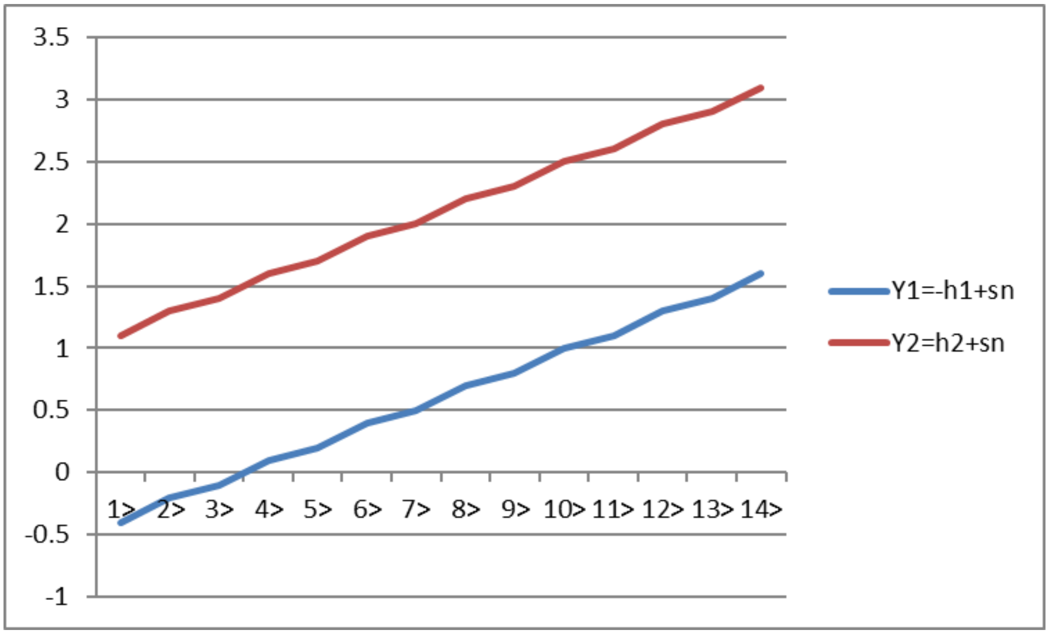
PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.75, $\alpha = 0.01$,



Sequential probabilities				
Lower prop. 0.01				
Higher prop. 0.5				
Alpha 1%				
Power 90%				
Experiments 14				
General Info				
Regression Coefficient: 0.14866				
Low Intercept: 0.49891				
High Intercept: 0.97926				
Sequential Probability Ratioos				
trial	lower limit		higher limit	
1>	-0.4	continue	1.1	continue
2>	-0.2	continue	1.3	63.80%
3>	-0.1	continue	1.4	47.50%
4>	0.1	2.40%	1.6	39.30%
5>	0.2	4.90%	1.7	34.50%
6>	0.4	6.60%	1.9	31.20%
7>	0.5	7.70%	2	28.90%
8>	0.7	8.60%	2.2	27.10%
9>	0.8	9.30%	2.3	25.70%
10>	1	9.90%	2.5	24.70%
11>	1.1	10.30%	2.6	23.80%
12>	1.3	10.70%	2.8	23%
13>	1.4	11%	2.9	22.40%
14>	1.6	11.30%	3.1	21.90%

After trial 14
Reject if:
- Less than 11.3% are positive
- More than 21.9 are positive
Otherwise continue
or accept no difference

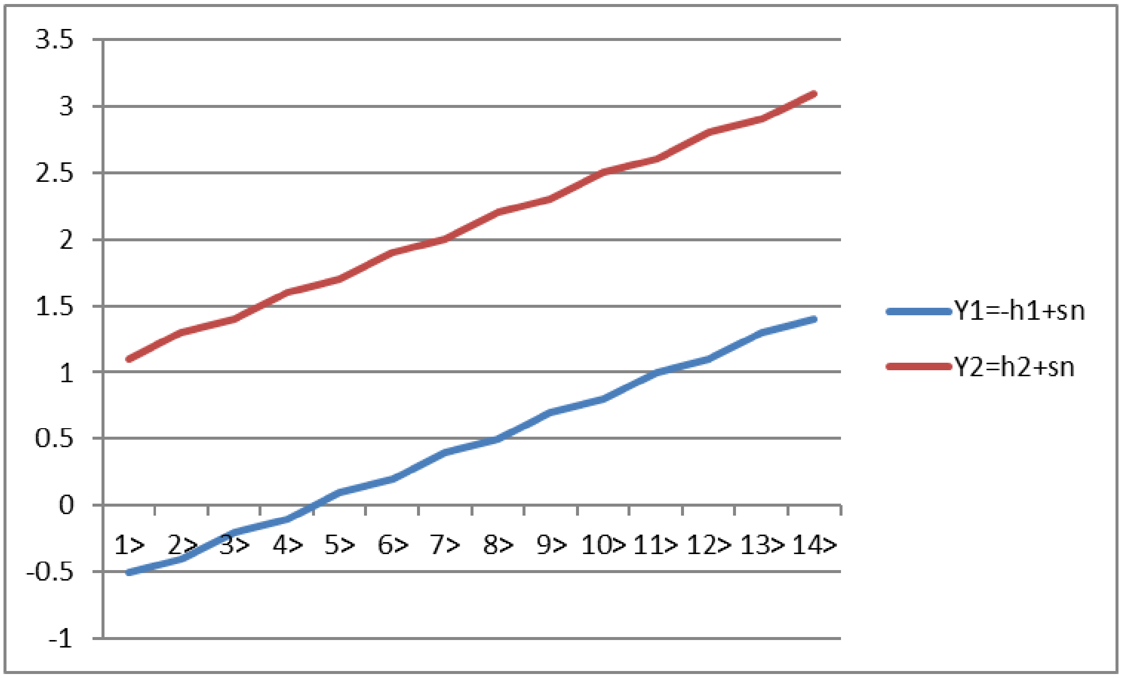
PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.90, $\alpha = 0.01$,



Sequential probabilities				
Lower prop. 0.01				
Higher prop. 0.5				
Alpha 1%				
Power 95%				
Experiments 14				
General Info				
Regression Coefficient: 0.14866				
Low Intercept: 0.64975				
High Intercept: 0.99102				
SEQUENTIAL PROBABILITY RATIO				
trial	lower limit		higher limit	
1>	-0.5	continue	1.1	continue
2>	-0.4	continue	1.3	64.40%
3>	-0.2	continue	1.4	47.90%
4>	-0.1	continue	1.6	39.60%
5>	0.1	1.90%	1.7	34.70%
6>	0.2	4%	1.9	31.40%
7>	0.4	5.60%	2	29%
8>	0.5	6.70%	2.2	27.30%
9>	0.7	7.60%	2.3	25.90%
10>	0.8	8.40%	2.5	24.80%
11>	1	9%	2.6	23.90%
12>	1.1	9.50%	2.8	23.10%
13>	1.3	9.90%	2.9	22.50%

14>	1.4	10.20%	3.1	21.90%
After trial 14				
Reject if:				
- Less than 10.2% are positive				
- More than 21.9% are positive				
Otherwise continue				
or accept no difference				

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.95, $\alpha = 0.01$,

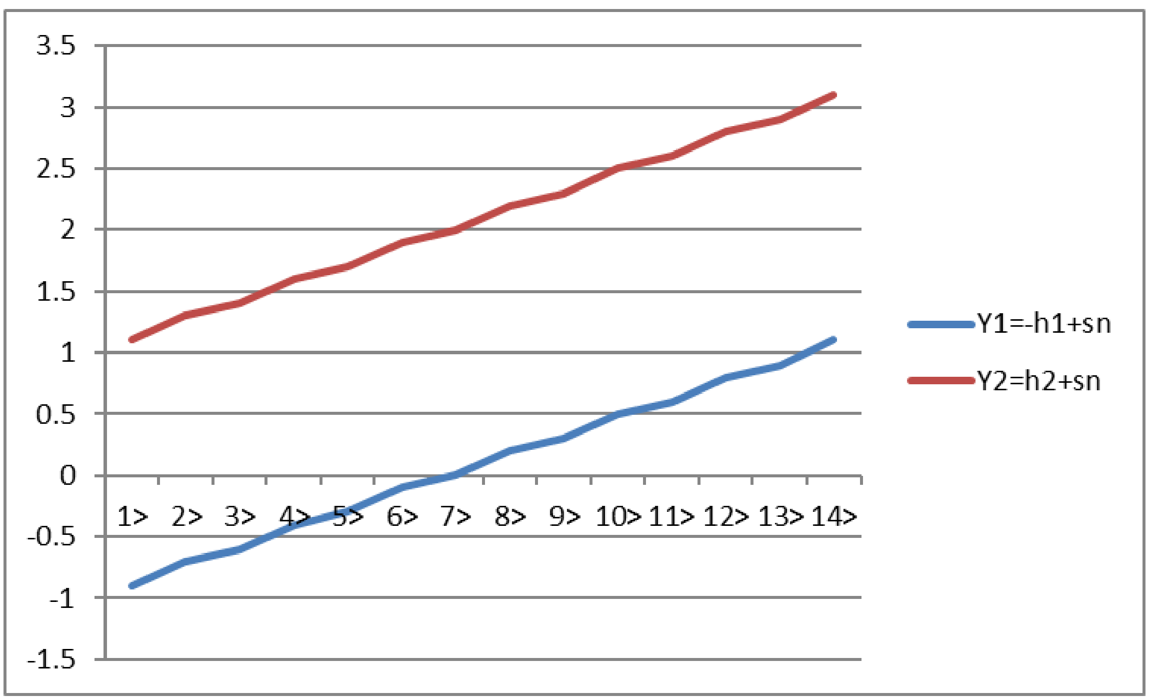


Sequential probabilities
Lower prop. 0.01
Higher prop. 0.5
Alpha 1%
Power 99%
Experiments 14

General Info				
Regression Coefficient: 0.14866				
Low Intercept: 1				
High Intercept: 1				
Sequential Probability Ratio				
trial	lower limit		higher limit	
1>	-0.9	continue	1.1	Continue

2>	-0.7	continue	1.3	64.90%
3>	-0.6	continue	1.4	48.20%
4>	-0.4	continue	1.6	39.90%
5>	-0.3	continue	1.7	34.90%
6>	-0.1	continue	1.9	31.50%
7>	0	0.60%	2	29.20%
8>	0.2	2.40%	2.2	27.40%
9>	0.3	3.80%	2.3	26%
10>	0.5	4.90%	2.5	24.90%
11>	0.6	5.80%	2.6	24%
12>	0.8	6.50%	2.8	23.20%
13>	0.9	7.20%	2.9	22.60%
14>	1.1	7.70%	3.1	22%
After trial 14				
Reject if:				
- Less than 7.7% are positive				
- More than 22% are positive				
Otherwise continue				
or accept no difference				

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.99, $\alpha = 0.01$,



DATE SET 2: t_i (i= 1, 2…,13): $\beta = 0.75, 0.90, 0.95, 0.99$, $\alpha = 0.02$, $p_1 = 0.01$, $p_2 = 0.05$

N	1	2	3	4	5	6	7	8	9	10	11	12	13
sample	384	1186	1471	2236	2772	2967	3812	4880	6104	6634	7229	8072	8484

Lower prop. 0.01
Higher prop. 0.5
Alpha 2%
Power 75%
Experiments 13

General Info

Regression Coefficient: 0.14866

Low Intercept: 0.29729

High Intercept: 0.78874

SEQUENTIAL PROBABILITY RATIO				
Trial	lower limit		higher limit	
1>	-0.1	continue	0.9	93.70%
2>	0	0%	1.1	54.30%
3>	0.1	5%	1.2	41.20%
4>	0.3	7.40%	1.4	34.60%
5>	0.4	8.90%	1.5	30.60%
6>	0.6	9.90%	1.7	28%
7>	0.7	10.60%	1.8	26.10%
8>	0.9	11.10%	2	24.70%
9>	1	11.60%	2.1	23.60%
10>	1.2	11.90%	2.3	22.80%
11>	1.3	12.20%	2.4	22%
12>	1.5	12.40%	2.6	21.40%
13>	1.6	12.60%	2.7	20.90%
After trial 13				
Reject if:				
- Less than 12.6% are positive				
- More than 20.9% are positive				
Otherwise continue				
or accept no difference				

Sequential probabilities

Lower prop. 0.01

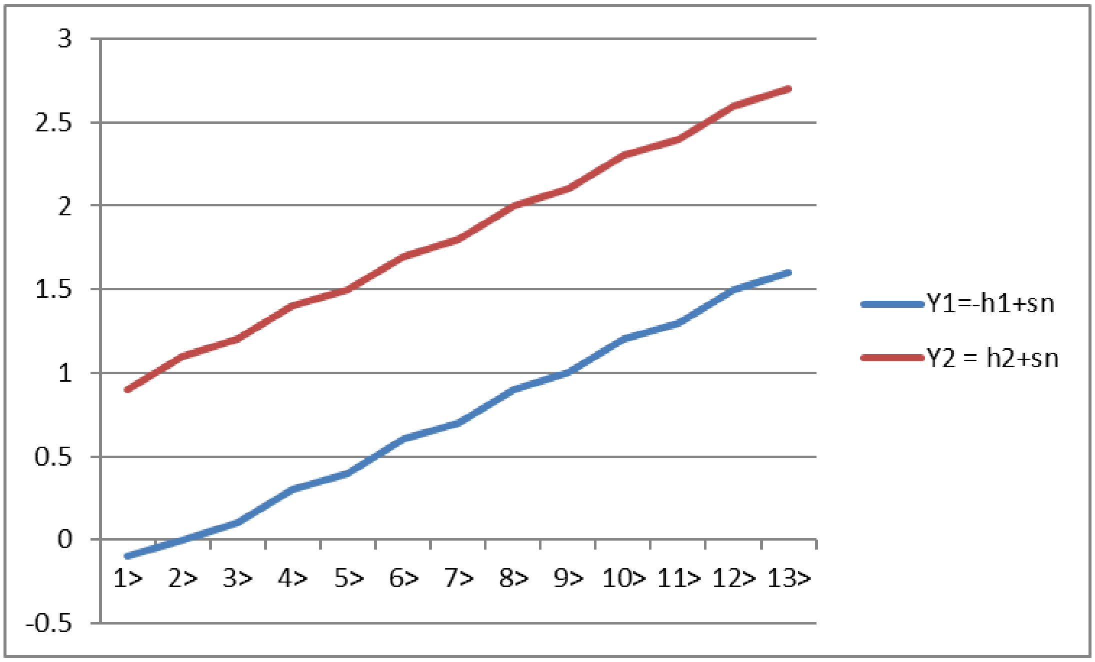
Higher prop. 0.5

Alpha 2%

Power 90%

Experiments 13

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.75, $\alpha = 0.02$,



Sequential probabilities

Lower prop. 0.01

Higher prop. 0.5

Alpha 2%

Power 90%

Experiments 13

General Info

Regression Coefficient: 0.14866

Low Intercept: 0.4967

High Intercept: 0.82841

SEQUENTIAL PROBABILITY RATIO				
trial	lower limit		higher limit	
1>	-0.3	continue	1	97.70%
2>	-0.2	continue	1.1	56.30%
3>	-0.1	continue	1.3	42.50%
4>	0.1	2.40%	1.4	35.60%
5>	0.2	4.90%	1.6	31.40%
6>	0.4	6.60%	1.7	28.70%
7>	0.5	7.80%	1.9	26.70%
8>	0.7	8.70%	2	25.20%
9>	0.8	9.30%	2.2	24.10%
10>	1	9.90%	2.3	23.10%
11>	1.1	10.40%	2.5	22.40%
12>	1.3	10.70%	2.6	21.80%
13>	1.4	11%	2.8	21.20%

After trial 13

Reject if:

- Less than 11% are positive
- More than 21.2% are positive

Otherwise continue

or accept no difference

Sequential probabilities

Lower prop. 0.01

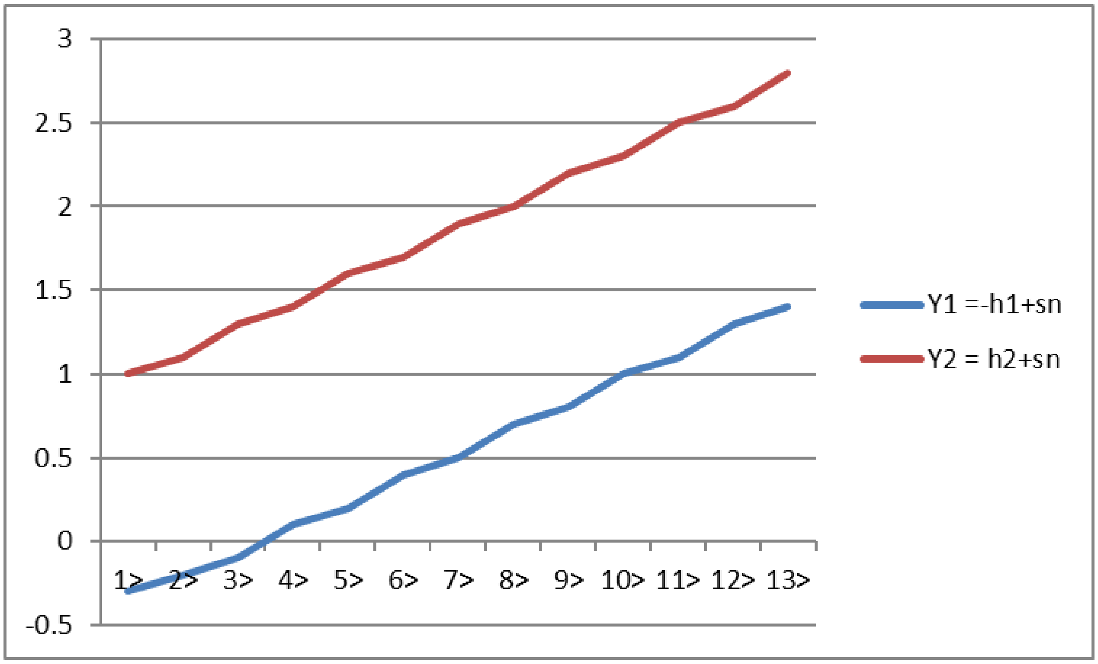
Higher prop. 0.5

Alpha 2%

Power 95%

Experiments 13

Performance of Sequential Probability Ratio Test β = 0.90, α= 0.02



Sequential probabilities

Lower prop. 0.01

Higher prop. 0.5

Alpha 2%

Power 95%

Experiments 13

General Info

Regression Coefficient: 0.14866

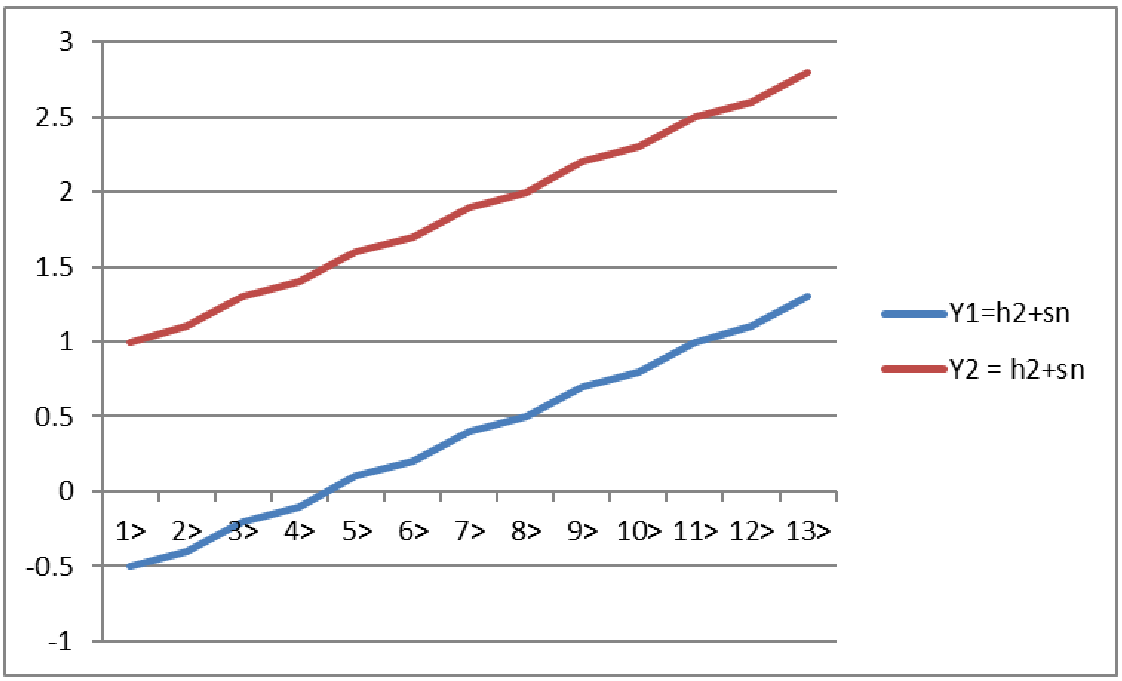
Low Intercept: 0.64754

High Intercept: 0.84018

SEQUENTIAL PROBABILITY RATIO				
trial	lower limit		higher limit	
1>	-0.5	continue	1	98.90%
2>	-0.4	continue	1.1	56.90%
3>	-0.2	continue	1.3	42.90%
4>	-0.1	continue	1.4	35.90%
5>	0.1	1.90%	1.6	31.70%
6>	0.2	4.10%	1.7	28.90%
7>	0.4	5.60%	1.9	26.90%
8>	0.5	6.80%	2	25.40%

9>	0.7	7.70%	2.2	24.20%
10>	0.8	8.40%	2.3	23.30%
11>	1	9%	2.5	22.50%
12>	1.1	9.50%	2.6	21.90%
13>	1.3	9.90%	2.8	21.30%
After trial 13				
Reject if:				
- Less than 9.9% are positive				
- More than 21.3% are positive				
Otherwise continue				
or accept no difference				

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.95, $\alpha = 0.02$



Sequential probabilities

Lower prop. 0.01

Higher prop. 0.5

Alpha 2%

Power 99%

Experiments 13

General Info

Regression Coefficient: 0.14866
Low Intercept: 0.99779
High Intercept: 0.84916

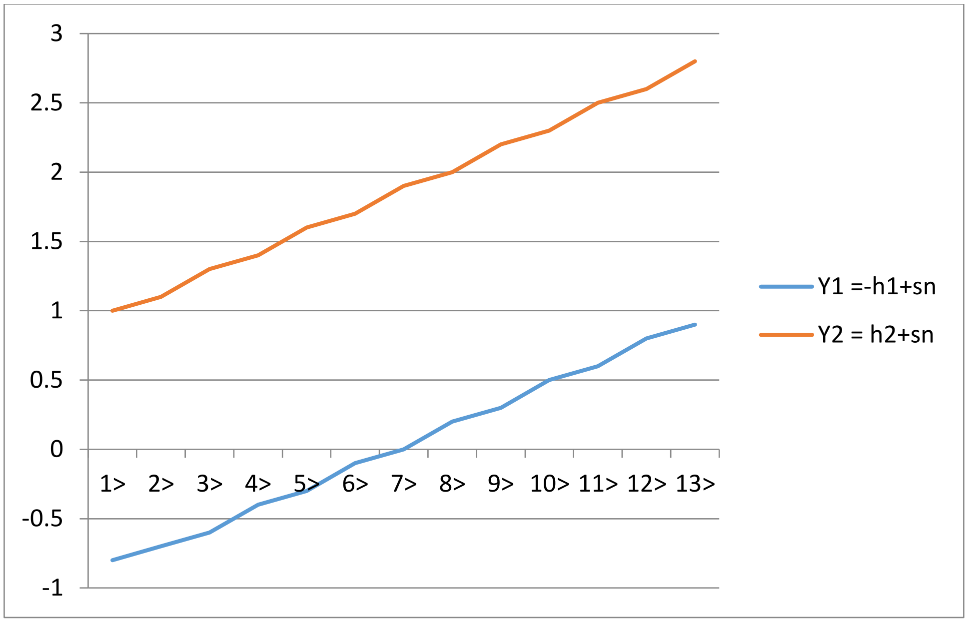
SEQUENTIAL PROBABILITY RATIO

trial	lower limit		higher limit	
1>	-0.8	continue	1	99.80%
2>	-0.7	continue	1.1	57.30%
3>	-0.6	continue	1.3	43.20%
4>	-0.4	continue	1.4	36.10%
5>	-0.3	continue	1.6	31.80%
6>	-0.1	continue	1.7	29%
7>	0	0.60%	1.9	27%
8>	0.2	2.40%	2	25.50%
9>	0.3	3.80%	2.2	24.30%
10>	0.5	4.90%	2.3	23.40%
11>	0.6	5.80%	2.5	22.60%
12>	0.8	6.60%	2.6	21.90%
13>	0.9	7.20%	2.8	21.40%

After trial 13
Reject if:
- Less than 7.2% are positive
- More than 21.4% are positive

Otherwise continue
or accept no difference

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.99, $\alpha = 0.01$



DATA SET 3: Release 4: t_i (i= 1, 2…,14): $\beta = 0.75, 0.90, 0.95, 0.99$ $\alpha = 0.05$ $p_1 = 0.01, p_2 = 0.05$

N	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
sample	254	788	1054	1393	2216	2880	3593	4281	5180	6003	7621	8783	9604	10064	10560

Lower prop. 0.01
Higher prop. 0.5
Alpha 5%
Power 75%
Experiments 15

General Info
Regression Coefficient: 0.14866
Low Intercept: 0.29053
High Intercept: 0.58933

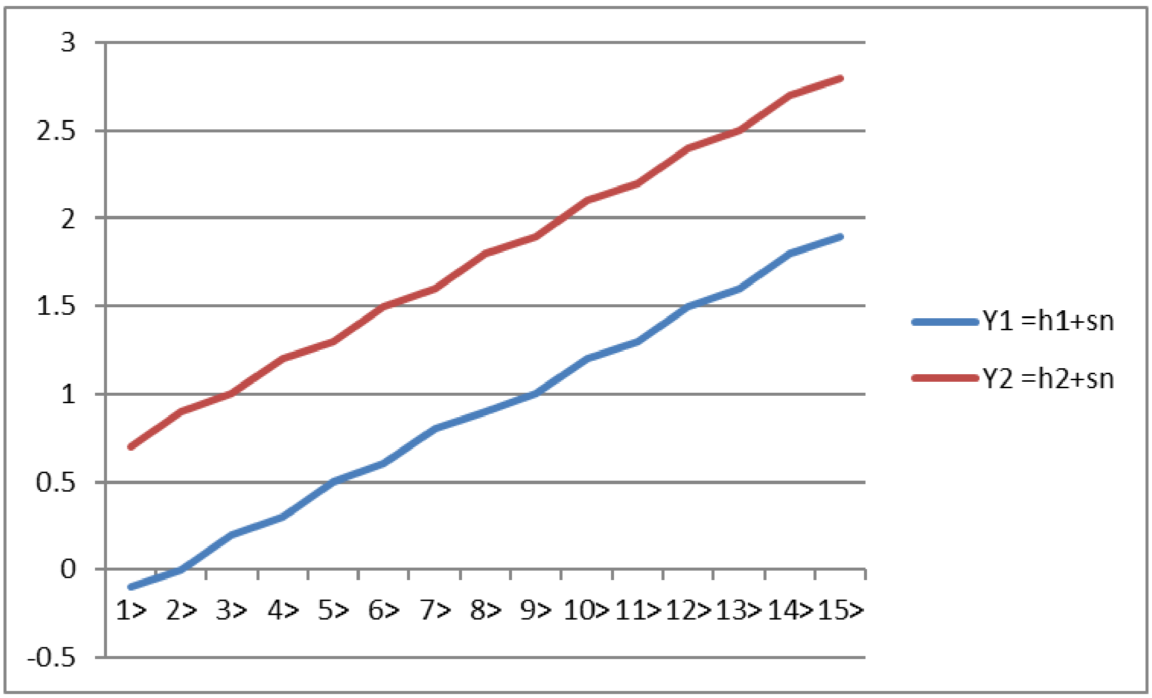
SEQUENTIAL PROBABILITY RATIO

trial	lower limit		higher limit	
1>	-0.1	continue	0.7	73.80%
2>	0	0.30%	0.9	44.30%
3>	0.2	5.20%	1	34.50%
4>	0.3	7.60%	1.2	29.60%
5>	0.5	9.10%	1.3	26.70%
6>	0.6	10%	1.5	24.70%
7>	0.8	10.70%	1.6	23.30%
8>	0.9	11.20%	1.8	22.20%
9>	1	11.60%	1.9	21.40%

10>	1.2	12%	2.1	20.80%
11>	1.3	12.20%	2.2	20.20%
12>	1.5	12.40%	2.4	19.80%
13>	1.6	12.60%	2.5	19.40%
14>	1.8	12.80%	2.7	19.10%
15>	1.9	12.90%	2.8	18.80%

After trial 15
Reject if:
- Less than 12.9% are positive
- More than 18.8% are positive
otherwise continue

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO(SPRT) β=0.75, $\alpha = 0.01$,

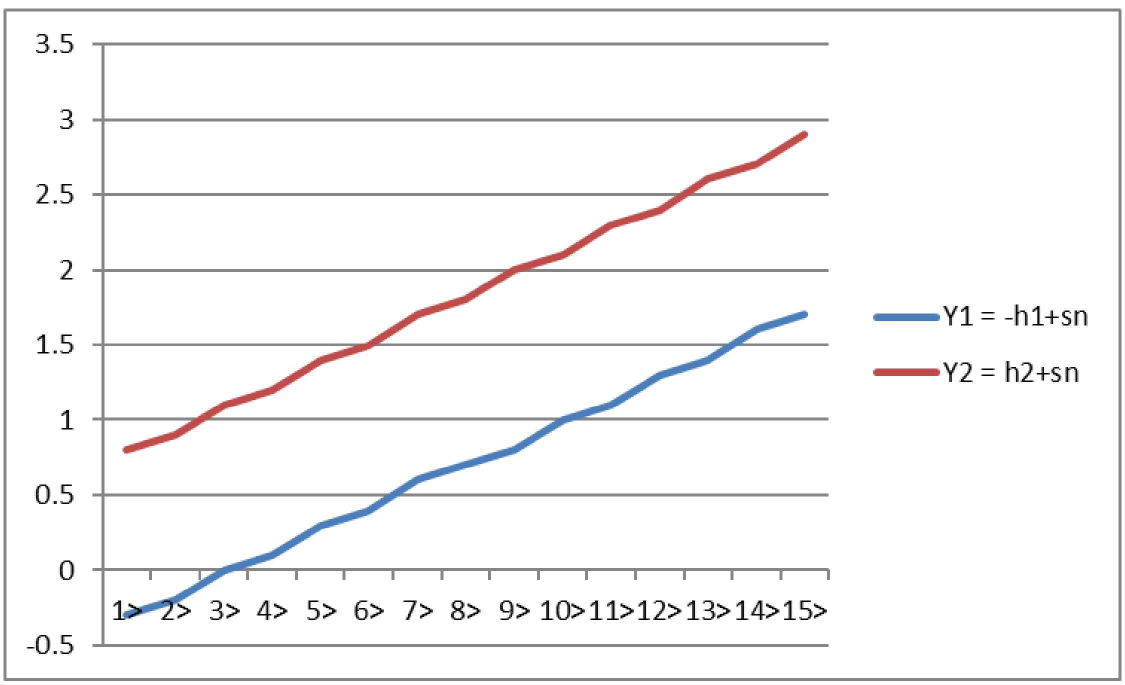


Sequential probabilities
Lower prop. 0.01
Higher prop. 0.5
Alpha 5%
Power 90%
Experiments 15

General Info
Regression Coefficient: 0.14866
Low Intercept: 0.48993

High Intercept: 0.62901

SEQUENTIAL PROBABILITY RATIO				
trial	lower limit		higher limit	
1>	-0.3	continue	0.8	77.80%
2>	-0.2	continue	0.9	46.30%
3>	0	continue	1.1	35.80%
4>	0.1	2.60%	1.2	30.60%
5>	0.3	5.10%	1.4	27.40%
6>	0.4	6.70%	1.5	25.30%
7>	0.6	7.90%	1.7	23.90%
8>	0.7	8.70%	1.8	22.70%
9>	0.8	9.40%	2	21.90%
10>	1	10%	2.1	21.20%
11>	1.1	10.40%	2.3	20.60%
12>	1.3	10.80%	2.4	20.10%
13>	1.4	11.10%	2.6	19.70%
14>	1.6	11.40%	2.7	19.40%
15>	1.7	11.60%	2.9	19.10%
After trial 15				
Reject if:				
- Less than 11.6% are positive				
- More than 19.1% are positive				
Otherwise continue				
or accept no difference				



Sequential probabilities

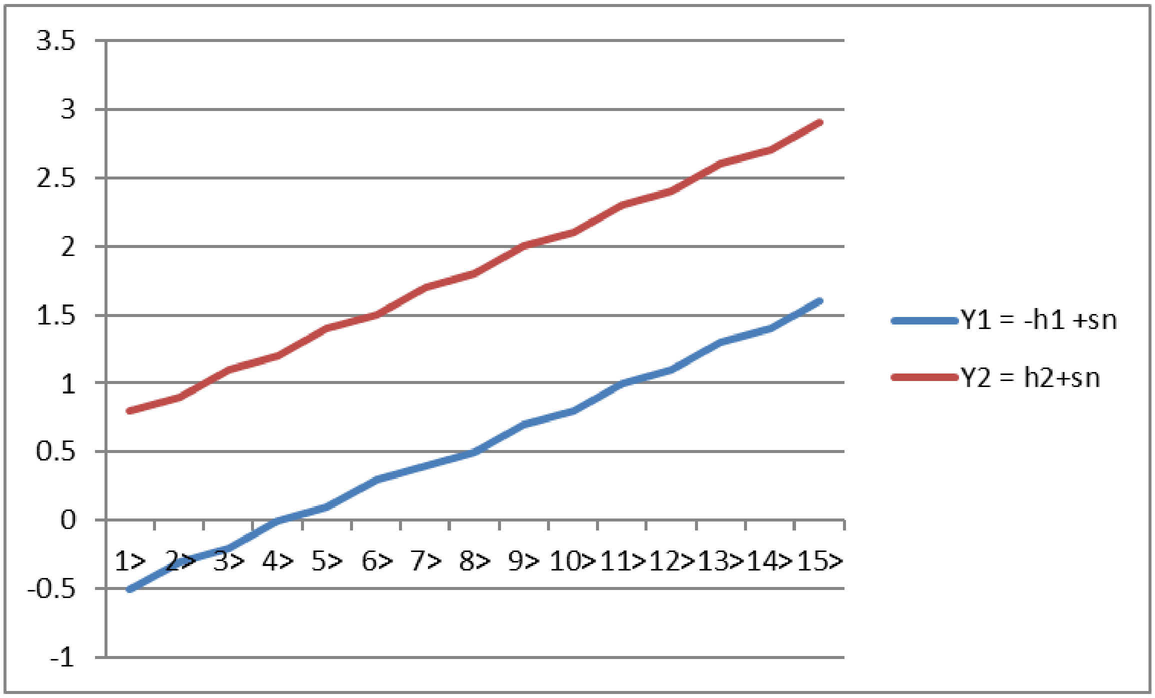
Lower prop. 0.01
Higher prop. 0.5
Alpha 5%
Power 95%
Experiments 15

General Info
Regression Coefficient: 0.14866
Low Intercept: 0.64078
High Intercept: 0.64078

SEQUENTIAL PROBABILITY RATIOOS

trial	lower limit		higher limit	
1>	-0.5	continue	0.8	78.90%
2>	-0.3	continue	0.9	46.90%
3>	-0.2	continue	1.1	36.20%
4>	0	continue	1.2	30.90%
5>	0.1	2.10%	1.4	27.70%
6>	0.3	4.20%	1.5	25.50%
7>	0.4	5.70%	1.7	24%
8>	0.5	6.90%	1.8	22.90%
9>	0.7	7.70%	2	22%
10>	0.8	8.50%	2.1	21.30%
11>	1	9%	2.3	20.70%
12>	1.1	9.50%	2.4	20.20%
13>	1.3	9.90%	2.6	19.80%
14>	1.4	10.30%	2.7	19.40%
15>	1.6	10.60%	2.9	19.10%
After trial 15				
Reject if:				
- Less than 10.6% are positive				
- More than 19.1% are positive				
Otherwise continue				
or accept no difference				

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.95, $\alpha = 0.05$,



Sequential probabilities
Lower prop. 0.01
Higher prop. 0.5
Alpha 5%
Power 99%
Experiments 15

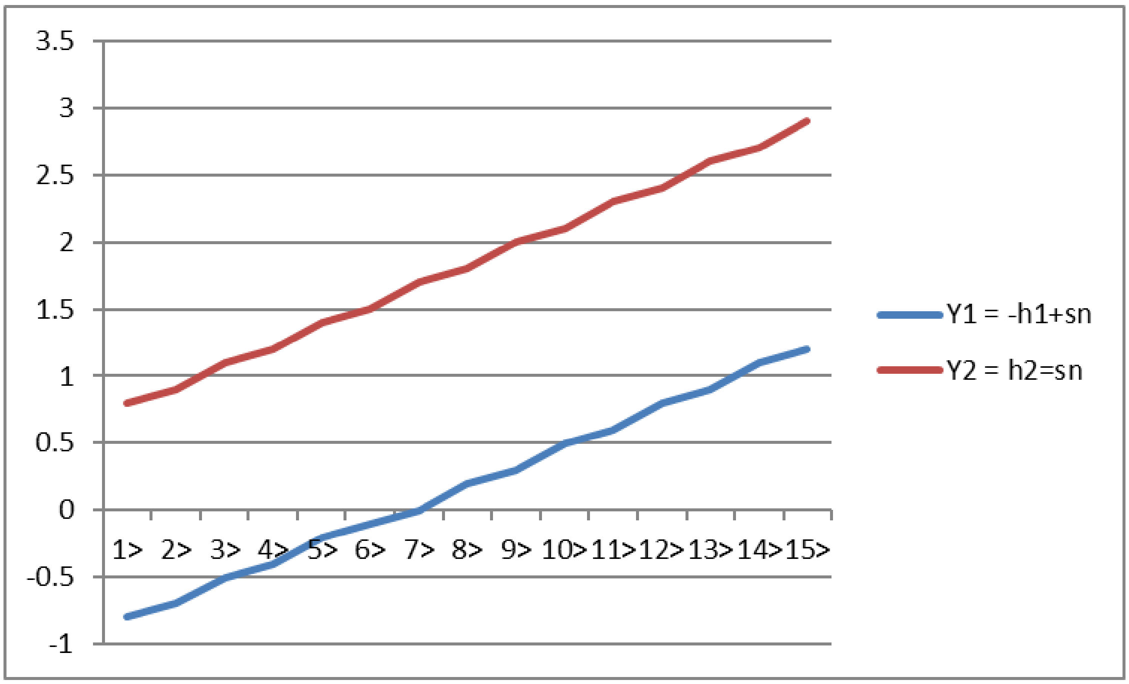
General Info
Regression Coefficient: 0.14866
Low Intercept: 0.99102
High Intercept: 0.64975

SEQUENTIAL PROBABILITY RATIO

trial	lower limit		higher limit	
1>	-0.8	continue	0.8	79.80%
2>	-0.7	continue	0.9	47.40%
3>	-0.5	continue	1.1	36.50%
4>	-0.4	continue	1.2	31.10%
5>	-0.2	continue	1.4	27.90%
6>	-0.1	continue	1.5	25.70%
7>	0	0.70%	1.7	24.10%
8>	0.2	2.50%	1.8	23%
9>	0.3	3.90%	2	22.10%
10>	0.5	5%	2.1	21.40%
11>	0.6	5.90%	2.3	20.80%
12>	0.8	6.60%	2.4	20.30%

13>	0.9	7.20%	2.6	19.90%
14>	1.1	7.80%	2.7	19.50%
15>	1.2	8.30%	2.9	19.20%
After trial 15				
Reject if:				
- Less than 8.3% are positive				
- More than 19.2% are positive				
Otherwise continue				
or accept no difference				

PERFORMANCE OF SEQUENTIAL PROBABILITY RATIO (SPRT) β=0.99, $\alpha = 0.05$,



CONCLUSION

We obtained the sequential probability ratio test for various values of β and α and the different data set experiment times with inverse Rayleigh predicting software distribution of a life truncated product. The proposed plan yields the minimum efficient until a decision is made on the lot or process sampled, after each item is taken, a decision is made to accept, reject or continue sampling

The proposed plan is useful in minimizing the both the producer's and consumer' risk. However, the decision on the single sampling approach can be reach at $d_1 + d_2 \leq c$ else the experimenter terminates the sample and makes decision, therefore the propose SPRT is more economical than the double acceptance sampling, it is also required minimum sample to reach a decision.

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