Unsteady MHD Flow Past an Accelerated Vertical Plate with Effect of Thermal Radiation

YALE IBRAHIM DANJUMA[1], ABUBAKAR SHEHU[2]
[1, 2] *Department of Mathematics, Kebbi State University of Science and Technology Aliero, Nigeria*

Abstract- An unsteady magneto-hydro-dynamics flow past an accelerated vertical plate with effects of thermal radiation has been carried out. The dimensionless governing equations are solved using Laplace transform technique. The flow characteristics solution of velocity, temperature and concentration field were obtained and analyzed for the different physical parameters like thermal Grashof Number, Modified Grashof Number, permeability parameter, Prandtl Number, Radiation parameter, Schmidt Number, and time. It is observed that the velocity profile increases with increasing parameters like, Sc, Gc, M, R and t. and also decrease with the decreases Pr and Gr. the temperature profile shows the decrease with the decreasing in Pr and t. and increase in R. While the concentration profile shows a decreasing profiles in Sc and t.

Indexed Terms- Unsteady, Accelerated, Vertical Plate, MHD and Radiation.

I. INTRODUCTION

Thermal radiation is an important factor in the thermodynamic analysis of many high temperature systems like solar collectors, boilers and furnaces. The simultaneous effect of heat and mass transfer in the presence of thermal radiation plays an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, cooling of towers, gas turbines and various propulsion device for aircraft, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. MHD plays an important role in agriculture, petroleum industries, geophysics and in astrophysics. MHD flow has application in metrology, solar physics and in motion of earth's core. Also it has applications in the field of stellar and planetary magnetospheres, aeronautics, chemical engineering and electronics. In the field of power generation, MHD is receiving considerable attention due to the possibilities it offers for much higher thermal efficiencies in power plants. [1] studied the thermal radiation effect on a transient MHD flow with mass transfer past an impulsively fixed infinite vertical plate. [2] carried out study on the analytical solution of MHD flow with radiation over a stretching sheet embedded in a porous medium. [3] investigated the radiative free convective non-Newtonian fluid flow past a wedge embedded in a porous medium. [4] investigated the combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium. [5] studied the thermal radiation effects on unsteady hydro magnetic gas flow along an inclined plane with indirect natural convection. Thermal radiation on linearly accelerated vertical plate with variable temperature and uniform mass flux [6]. Radiation and mass transfer effects on two- dimensional flow past an impulsively started infinite vertical plate [7]. The study on the radiation effects on MHD combined convection and mass transfer flow past a vertical porous plate embedded in a porous medium with heat generation was carried out by [8]. Unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical porous plate with variable viscosity and thermal conductivity [9]. Radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation was investigated by [10].

II. FORMULATION OF THE PROBLEM

The unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature in the presence of MHD and thermal Radiation is considered. The flow is assumed to be in x-direction which takes vertical plate in the upward direction. The y-axis is taken to be normal to the plate. Initially the plate and the fluid are in same temperature } with the same concentration level $C^{'}$ at all points. At time $t^{'}$ > 0 the plate accelerated with velocity $U = \frac{t^{'} u_{\circ}^{2}}{v}$ in its own plane. The plate temperature is raised to $T_{w}^{'}$ and the level of concentration near the plate is raised to $C_{w}^{'}$ linearly with the time t.

III. GOVERNING EQUATION

By Boussinesq's approximation the unsteady flow is governed by the following equation.

$$\frac{\partial u}{\partial t^{'}} = g\beta(T - T_{\infty}) + g\beta^{*}(C^{'} - C^{'}) + v\frac{\partial^{2} u}{\partial y^{2}} - \frac{\sigma\beta_{\circ}^{2} u^{'} u_{\circ}^{2}}{\rho v}$$

(1)

$$\rho C_{P}\frac{\partial T}{\partial t^{'}} = k\frac{\partial^{2} T}{\partial y^{2}} - \frac{\partial q_{r}}{\partial y^{'}} \quad (2)$$

$$\frac{\partial C^{'}}{\partial t^{'}} = D\frac{\partial^{2} C^{'}}{\partial y^{2}} \quad (3)$$

Where (1) is the momentum equation (2) is the energy equation and (3) is the mass concentration equation. Where U is the velocity of the fluid, T is the fluid temperature, C' is the concentration g is gravitational constant, β and β^{*} are the thermal expansions of fluid and concentration, t' is the time, ρ is the fluid density, C ρ is the specific heat capacity v is the viscosity of the fluid, k is the thermal conductivity of the fluid.

The initial and boundary condition are;

$$u = 0, \quad T = T_{\infty,} \; C^{'} = C_{\infty}^{'},$$

$$for \quad all \quad y, t^{'} \le 0$$

$$t^{'} \ge 0: \; u = u_{\circ} t, T = T_{\infty} + (T_{w} - T_{\infty}) at C^{'} = C_{w} at \;\; y = 0$$

$$u \to 0, T \to T_{\infty,} C^{'} \to C^{'}_{\infty}, \quad at \quad y \to \infty$$

(4)

Where $A = \left(\frac{u_{\circ}^{2}}{v}\right)^{\frac{1}{3}}$, A is a constant

$$\frac{\partial q_{r}}{\partial y^{'}} = -4a^{*}\sigma\left(T^{'4}_{\infty} - T^{'4}\right) \quad (5)$$

It is expected that the temperature differences within the flow are sufficiently small such that $T^{'4}$ may be expressed as a linear function of the temperature. This is accomplished by expanding $T^{'4}$ in a Taylor series about T'_{∞} and neglecting higher order terms, thus

$$T^{'4} \cong 4T^{'3}_{\infty} T' - 3T^{'4}_{\infty} \quad (6)$$

By using equations (5) and (6), equation (2) reduces to;

$$\rho C_{p}\frac{\partial T'}{\partial t'} = k\frac{\partial^{2} T'}{\partial y'^{2}} + 16a^{*}\sigma T_{\infty}^{'3}\left(T_{\infty}^{'} - T^{'}\right)$$

(7)

The non-dimensional quantities are:

$$U = \left(\frac{u^{'}}{u_{\circ}}\right), t = \left(\frac{t^{'} u_{\circ}^{2}}{v}\right), Y = \left(\frac{y u_{\circ}}{v}\right), \theta = \frac{T^{'} - T^{'}_{\infty}}{T^{'}_{w} - T^{'}_{\infty}},$$

$$Gr = \frac{g\beta v\left(T_{w} - T_{\infty}\right)}{u_{\circ}^{3}}, \quad C = \frac{C^{'} - C^{'}_{\infty}}{C^{'}_{w} - C^{'}_{\infty}},$$

$$Gc = g\beta^{*} v\frac{\left(C^{'}_{w} - C^{'}_{\infty}\right)}{u_{\circ}^{3}}, \Pr = \frac{\mu c_{\rho}}{k}, Sc = \frac{v}{D},$$

$$M = \frac{\sigma\beta_{\circ}^{2} u^{'}}{\rho}, \quad R = \frac{16a^{*} v^{2}\sigma T_{\infty}^{'3}}{k u_{\circ}^{'2}} \quad (8)$$

The non-dimensional quantities of equation (9) which analyzed (1) to (4) and they lead to the dimensionless equations are as follows;

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^{2} U}{\partial Y^{2}} - MU \quad (9)$$

$$\frac{1}{\Pr}\frac{\partial\theta}{\partial t} = \frac{\partial^{2}\theta}{\partial Y^{2}} + R\theta \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^{2} C}{\partial Y^{2}} \quad (11)$$

where Sc is the Schmidt number, Pr is the Prandtl number and Gr and Gc are the Grashof numbers.

The initial and boundary conditions in dimensionless form are:

$U = 0, \theta = 0, C = 0,$ for all Y, $t \le 0$

$t > 0: U = 1, \theta = 1, C = 1, at Y = 0$

$$U \to 0, \theta \to 0, C \to 0, as, Y \to \infty \qquad (12)$$

IV. SOLUTION TO THE PROBLEM

Equation (9) to (11) are solved subject to the boundary conditions of (12) and the solutions are obtained for velocity, temperature and concentration field in terms of exponential and complementary error function using Laplace transform technique as follows;

$$C = erfc\left(\eta\sqrt{Sc}\right) \qquad (13)$$

$$\theta = \frac{1}{2}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} + \sqrt{Rt}\right) + e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} - \sqrt{Rt}\right)\right\}$$

(14)

$$U = \frac{1}{2}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) + e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right\}$$

$$\frac{Grt}{2a}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) + e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right] -$$

$$\frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) - e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right] +$$

$$\frac{Gr}{2f}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) + e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right\} +$$

$$\frac{Gct}{2h}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) + e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right] -$$

$$\frac{\eta\sqrt{t}}{\sqrt{M}}\left[e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) - e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right] -$$

$$\frac{Gc}{2M}\left\{e^{2\eta\sqrt{mt}} erfc\left(\eta + \sqrt{Mt}\right) + e^{-2\eta\sqrt{mt}} erfc\left(\eta - \sqrt{Mt}\right)\right\} -$$

$$\frac{Grt}{2a}$$

$$\left[e^{-2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr} - \sqrt{Rt}\right) + e^{2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr} + \sqrt{Rt}\right)\right] -$$

$$\frac{\eta\sqrt{\Pr t}}{\sqrt{R}}\left[e^{-2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr} - \sqrt{Rt}\right) - e^{2\eta\sqrt{\Pr Rt}} erfc\left(\eta\sqrt{\Pr} + \sqrt{Rt}\right)\right] +$$

$$\frac{Gr}{2f}\left\{e^{2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} + \sqrt{Rt}\right) + e^{-2\eta\sqrt{Rt}} erfc\left(\eta\sqrt{\Pr} - \sqrt{Rt}\right)\right\} -$$

$$\frac{Gct}{h}\left\{\left(1 + 2\eta^2 Sc\right) erfc\left(\eta\sqrt{Sc}\right) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}} e^{-\eta^2 Sc}\right\} -$$

$$\frac{Gc}{M}\left[erfc(\eta)\sqrt{Sc}\right] \qquad (15)$$

Where

$$a = (\Pr - 1), \quad h = (Sc - 1), \quad f = (R - M),$$

$$\eta = \frac{y}{2\sqrt{t}}$$

V. RESULT AND DISCUSSION

The solution obtained in this problem for the flow characteristics of velocity, temperature and concentration fields. The solution where obtained by Laplace transform and the results are displayed inform of graphs are presented. To interpret the results for a better understanding of the problem numerical computations are carried out for different physical parameters like Pr, Gr, Gc, Sc, R, M and t upon the nature of the flow and transport. The value of the Schmidt number Sc is taken to be 0.57, which corresponds to water-vapor. The value of the Prandtl number Pr is chosen such that it represents air (Pr=0.71).

The effects of velocity for different values of thermal Grashof number Gr is presented in figure 4.1. It shows that velocity increases with increasing values of Gr. The effect of velocity for different values of time t is presented in figure 4.2. It shows that velocity increases with increasing values of t. The effects of velocity for different values of mass Grashof number Gc. is presented in figure 4.3. It observed that velocity increases with increasing values of Gc. The effect of velocity for different values of Schmidt number Sc is seen in figure 4.4. It was observed that velocity increases with increasing values of Sc. The effects of velocity for different values of magnetic field parameter M. is seen in figure 4.5. It was observed that velocity increases with increasing values of M. The effect of velocity for different values of Radiation Parameter R. is seen in figure 4.6. It was observed that velocity increases with increasing values of thermal radiation parameter. The effects of velocity for different values of Prandtl number Pr. is seen in figure 4.7. It was observed that the velocity is decreases with decreasing values of Pr.

The effects of temperature for different values of thermal radiation parameter R. is presented in figure 4.8. It shows that temperature rises increases with increasing values of R. The effect of temperature for different values of time t is presented in figure 4.9. It shows that temperature increases with increasing values of t. The effects of temperature for different values of Prandtl number Pr. is seen in figure 4.10. It was observed that the velocity is decreases with decreasing values of Pr.

The effects of concentration for different values of time t. is seen in figure 4.11. It was observed that the concentration increases with increasing values of t. The effects of concentration for different values of Schmidt number Sc. is seen in figure 4.12. It was observed that the concentration is decreases with decreasing values of Sc.

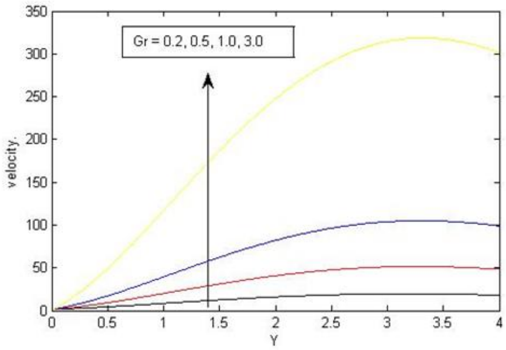

Figure 4.1 Velocity for different values of Gr

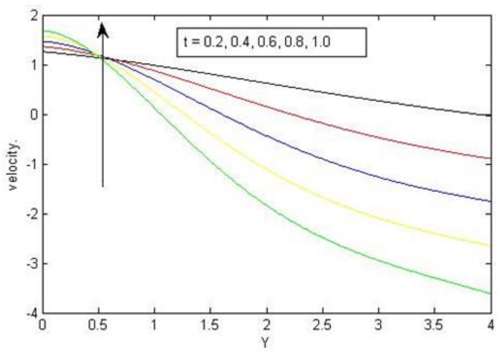

Figure 4.2 Velocity for different values of t.

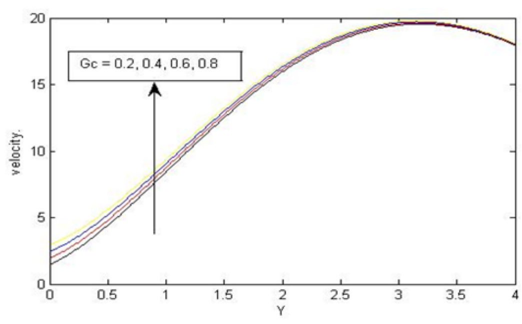

Figure 4.3 velocity for different values of Gc.

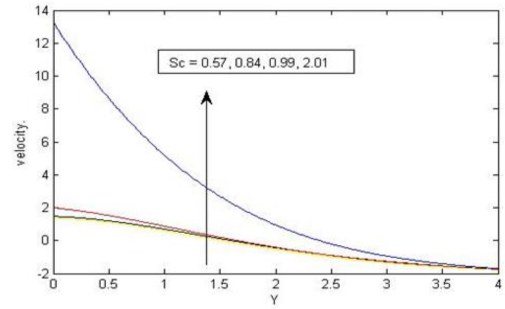

Figure 4.4 velocity for different values of Sc.

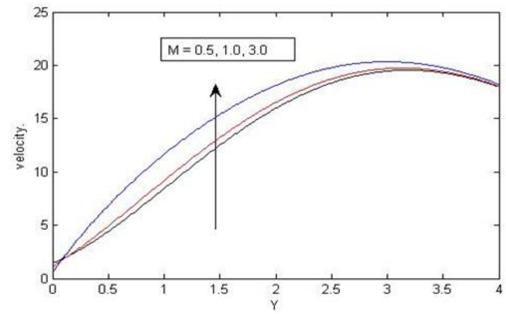

Figure 4.5 velocity for different values of M.

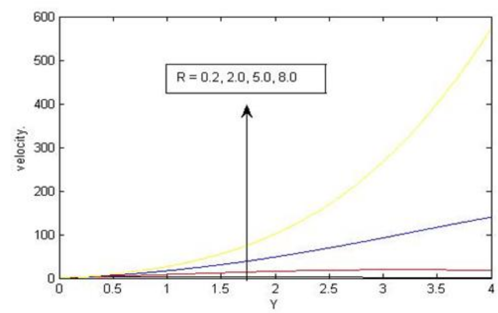

Figure 4.6 velocity for different values of R.

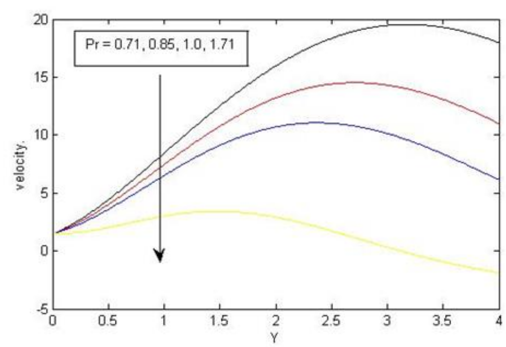

Figure 4.7 velocity frofle for different values of Pr.

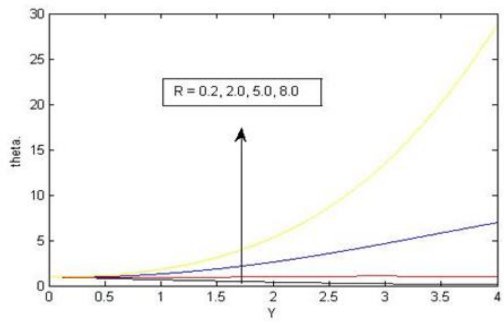

Figure 4.8 temperature for different of R.

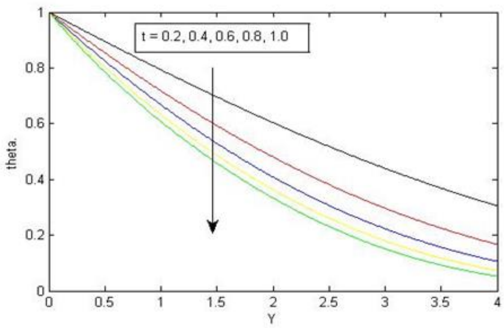

Figure 4.9 temperature for different values of t.

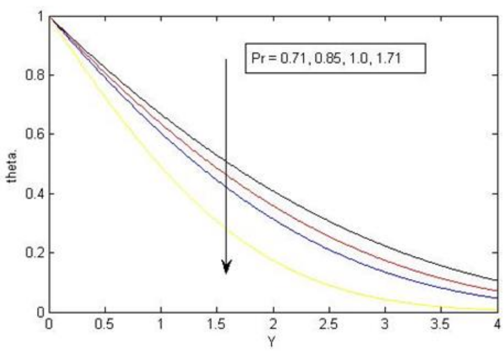

Figure 4.10 temperature for different values of Pr.

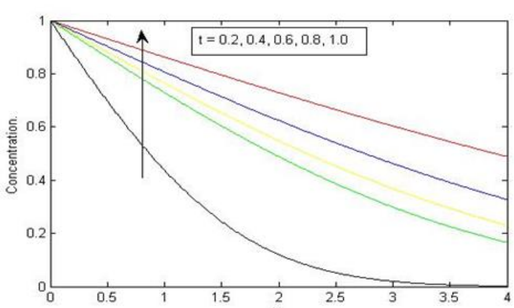

Figure 4.11 concentration for different values of t.

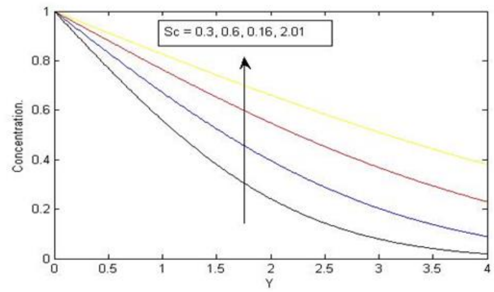

Figure 4.12 concentration for different values of Sc.

CONCLUSION

Unsteady MHD flow past an accelerated vertical plate with effect of thermal radiation have been studied, the dimensional governing equations are solved by Laplace-transform technique and computed for different parameters using MATLAB. The effect of different parameters such as thermal Grashof number, Schmidt number, Prandtl number, mass Grashof number, parameter, radiation parameter, magnetic field parameter and t are studied graphically. It was observed that the velocity profiles increases with the increasing parameters like Gr, t, Gc, Sc, M and R. It also observed that the temperature increases with the increasing in R and t, while decreases with the decreasing value of Pr. The concentration field increases with the increasing in time t, while decreases with the decreasing value of Sc.

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