The New General Integral Transform Decomposition Method for Solving Nonlinear Volterra Integral Equation Based On Numerical Formula

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Abstract- In this paper, we propose a new method, namely New GeneralIntegral Decomposition method (GIDM) for solving nonlinear Volterraintegral equations. This method is a combination of the new generalintegral transform method and decomposition method. The nonlinearterms can be easily handled by the use of Adomian polynomials. Thetechnique is described and illustrated with some examples. The resultsreveal that the proposed method is very efficient, simple and can beapplied to other nonlinear problems.

Indexed Terms- New General integral decomposition method, Nonlinear integral equations, Adomian polynomials, Noise terms phenomena.

I. INTRODUCTION

For solving nonlinear functional equation, Adomian decomposition method was introduced by George Adomian in 1980[3]. This technique provides an infinite series solution of the equation and nonlinear term is decomposed into an infinite series Adomian poynomials. Several linear and nonlinear ordinary ,partial and stochastic differential equations are solved easily by Adomian decomposition method . In this work , New general integral transform technique in combination with Adomian decomposition method is presented and modified This article considers the effectiveness of the new general integral decomposition method (NGDM)in solving nonlinear equations. In 2020 Jafari H.[1] introduced a new integral transform, named the the new general integral transform. In the present paper we focus solve nonlinear Volterra integral equations. Nonlinear Volterra integral equation arise in many scientific fields such as the Popukation dynamics, spread of epidemics and semiconductor devices. In this paper wehavefollowed the combined New general 1integral transform and Adomian decomposition method but while decomposing the nonlinear terms using decomposing the nonlinear term using Adomian polynomials, we have substituted the term *ui*with Newton Raphson formula. As we know that Newton Raphson formula is used for finding the better approximate solution of real valued function.

II. PRELIMINARIES

Definition 2.1: The New General Integral Transform.

Let $f(t)$ be a integrable function defined for $t\geq 0, p(s)\neq 0$ and $q(s)$ are positive real functions,we define the general integral transform $T(s)$ of $f(t)$ by the formula

$$T[f(t):s]=T(s)=p(s)\int_0^\infty f(t)e^{-q(s)t}dt \quad (1)$$

provided the integral exists for some q(s).

Definition 2.2: Nonlinear Volterra integral equation of the second kind.

Consider the following nonlinear Volterra integral equation with difference kernel i.e. $K(x, t) = k(x-t)$ defined as

$$u(x)=f(x)+\int_0^x k(x-t)\,F(u(t))dt \quad (2)$$

Where$f(x)$ is known real valued function and $F(u(t))$ is the nonlinear function of $u(t)$.

Apply New General integral transform on both sides of (2).After that using the linear property and convolution theorem of New General integral transform, we have

$T[u(x)]=T[f(x)]+T[k(x-t)]T[F(u(t))]$ (3)

The Methodology consists of approximating the solution of (2) as an infinite series given by

$u(x)=\sum_{n=0}^{\infty} un(x)$ (4)

However,the nonlinear term $F(u(x))$is decomposed as

$F(u(x))=\sum_{n=0}^{\infty} An(x)$ (5)

Where$A'^{n}s$ are modified Adomian polynomials which are based on newton Raphsonformulagiven as

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\ F\left(\sum_{i=0}^{n}\lambda^i\left(u_i - \frac{F(u_i)}{F'(u_i)}\right)\right) \quad (6)$$

Substituting equation (4) and (5) into (3), we get

$$T[\sum_{n=0}^{\infty} un(x)] = T[f(x)] + T[k(x-t)]T[\sum_{n=0}^{\infty} An(x)] \quad (7)$$

Using the linearity property of new general integral transform, we get

$$[\sum_{n=0}^{\infty} T[un(x)]] = T[f(x)] + T[k(x-t)][\sum_{n=0}^{\infty} T[An(x)]] \quad (8)$$

To determine the terms $u_{0(x)}, u_{1(x)}, u_{2(x)}, \ldots$of infinite series, comparing both sides of (8), we have the following iterative method

$$T[u_0(x)] = T[f(x)] \quad (9)$$

In general, the relation is given by

$$T[u_{n+1}(x)]=T[k(x-t)][\sum_{n=0}^{\infty} T[An(x)]] \quad (10)$$

Applying the inverse general transform to (9) and (10) we get,

$$u_0(x) = T^{-1}[T[f(x)]] \quad (11)$$

$$[u_{n+1}(x)] = T^{-1}\ [T[k(x-t)]T[An(x)]] \quad (12)$$

Adapting the value of$u_0(x)$ into (6) gives the value of $A0$ and then using the general iterative relation (12, we get values of $u1(x)$,$u_2(x)$,$u3(x)$,...and so on , which finally gives solution (4) to the given Volterra integral equation. The effectiveness of this technique for solving Volterra integral equations is shown by following numerical examples. Here we have also find the maximum absolute error estimation to show the adequacy of technique given as

$$ej = Max|u_{ex} - u_{apv}|$$

Wheree_j denotes the maximum absolute error at some x_j in the given interval.

Example 2.1 *Consider the following Volterra integral equation*

$$u(x) = x + \int_0^x u^2(t)dt$$

Which has the exact solution as$u(x) = tan(x)$.

Solution: WeTaking General integral transform on both side of equation (13)

And using the linearity property of general integral transform, we have

$$T[u(x)] = T[\ x] + T\left[\int_0^x u^2(t)dt\ \right] \quad (14)$$

That is

$$T[u(x)] = \frac{p(s)}{q(s)^2} + \frac{1}{q(s)}T[u^2(t)] \quad (15)$$

Using above technique, we have

$$T\left[\left[\sum_{n=0}^{\infty} un(x)\right]\right] = \frac{p(s)}{q(s)^2} + \frac{1}{q(s)}T[\sum_{n=0}^{\infty} T[An(x)]]$$

(16)

Where the nonlinear term $F(u(x)) = u_2(x)$ is decomposed using the formulagiven by (6). Certain terms of modified Adomian Polynomials are as follows:

$$A0 = \left(\frac{1}{2}\right)^2 u_0^2$$
$$A_1 = \left(\frac{1}{2}\right)^2 u_0 u_1$$
$$A2 = \left(\frac{1}{2}\right)^2 (2u_0u_2 + u_1^2)$$
$$A3 = \left(\frac{1}{2}\right)^2 (2u_0u_3 + 2u_1u_2)$$

Comparing both side of equation (16) gives,

$$T[u_0(x)] = \frac{p(s)}{q(s)^2} \qquad (17)$$

In general,

$$T[u_{n+1}(x)] = \frac{1}{q(s)} T[A_n(x)] \qquad (18)$$

Applying inverse General integral transform on both side of (17), gives

$u0(x) = x$,(19)

Use general relation, we have

$$u1(x) = \frac{x^3}{12} \qquad (20)$$

Continuing in this manner, we get

$$u_2(x) = \frac{x^5}{420},$$

$$u_3(x) = \frac{11X^7}{20160},$$

$$u_4 = \frac{x^9}{13440}$$

Subsequently, the approximate solution becomes

$$u(x) = x + \frac{x^3}{12} + \frac{x^5}{420} + \frac{11X^7}{20160} + \frac{x^9}{13440} + \cdots.$$

The exact solution and obtained by our technique corresponding to distinct values of x are presented in table 1 and exhibited through figure 1. The absolute error laid out in the table observed that the solutions are very much closed to the exact solution and the maximum absolute error is 0.0002.

XExact Solution Approximate Solution Absolute Error
0 00 0.000
0.01 0.010000333 0.01000008332.4967E-07
0.02 0.020002667 0.02000066662.0002E-06
0.00.030009003 0.03000226476.7382E-06
0.04 0.040021347 0.04000533371.6013E-05
0.050.0500417080.0500104179 3.129003E-05
0.06 0.060072104 0.06001800325.410076E-05
0.07 0.070114558 0.07002859038.5698966E-05
0.08 0.080171105 0.08004268031.2842467E-04
0.09 0.090243790 0.09006077461.8301537E-04
0.1 0.100334672 0.1000833752.530921E-04

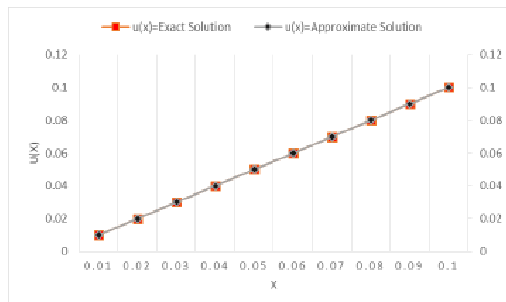

Fig. 1 – Comparison of Exact Solution and Approximate solution.

Example 2.2 *Consider the following Volterra integral equation [21]*

$$u(x) = 2x - \frac{x^4}{12} + \frac{1}{4}\int_0^x (x-t)u^2(t)\, dt \qquad (21)$$

Which has the exact solution as $u(x) = 2x$.

Solution.Taking General integral transform on both side of equation (21) and using the linearity property of general integral transform, we have

$$\text{T}\,[u(x)] = T[2x - \frac{x^4}{12} + \frac{1}{4}\int_0^x (x-t)u^2(t)\, dt]$$

$$T[u(x)] = T[2x - \frac{x^4}{12}] + \frac{1}{4}T[\int_0^x (x-t)u^2(t)\, dt] \qquad (22)$$

Using above technique, we have

$$T[[\sum_{n=0}^{\infty} un(x)]] =$$
$$T\left[2x - \frac{x^4}{12}\right] + \frac{1}{4}T[\sum_{n=0}^{\infty} T[An(x)]] \quad (23)$$

Comparing both side of equation (23) gives,

$$T[u_0(x)] = T\left[2x - \frac{x^4}{12}\right] \quad (24)$$

In general relation, we have

$$T\,[u_{n+1}(x)] = \frac{1}{p(s)}\,T[x]\,T[\sum_{n=0}^{\infty} An(x)\,] \quad (25)$$

Applying inverse General integral transform on both side we give

$$u_0(x) = \left[2x - \frac{x^4}{12}\right] \quad (26)$$

Use general relation, we have

$$u_1(x) = \frac{x^4}{48} - \frac{x^7}{2028} + \frac{x^{10}}{207360} \quad (27)$$

Continuing in this way, we get

$$u_2(x) = -\frac{x^6}{4777574399} + \frac{x^{13}}{905748480} - \frac{11x^{10}}{29030399} + \frac{x^7}{8065}$$
(28)

The approximate solution becomes,

$$u(x) = 2x - \frac{x^4}{16} - \frac{x^7}{2688} + \frac{x^{10}}{967680} + \frac{x^{13}}{905748480} - \frac{11x^{16}}{4777574399} + \cdots$$

The numerical results shown in table

x Exact Solution Approximate Solution Absolute Error
0 0 0 0.000
0.05 0.1 0.09999603.9063E-07
0.1 0.2 0.19999376.2500E-06
0.15 0.3 0.299968353.1641E-05
0.2 0.4 0.3999899991.0000E-04
0.25 0.50.499755832.4416E-04
0.3 0.60.599493665.0633E-04
0.35 0.7 0.69906189.813E-04
0.40 0.80.798399391.6006E-04
0.45 0.9 0.89743572.5643E-03
0.5 10.996090843.9092E-03

Maximum absolute error is 0.0030

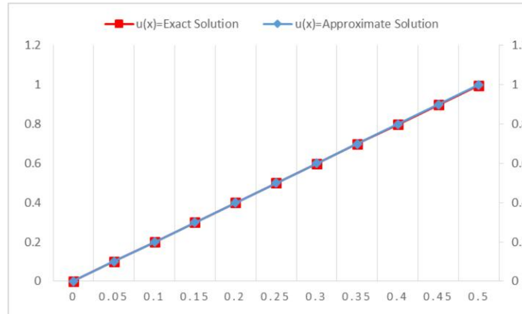

Fig. 2 – Comparison of Exact Solution and Approximate solution.

CONCLUSION

In this paper we used combination of Adomian Polynomials method and general integral

Transform for solution of nonlinear integral equation as demonstrated through solution

Tables and their graphs, it is observed that the approximate solution obtained by this technique is more accurate and error is minimum. The technique used in the paper is easy to implement and provides more accurate solution for nonlinear integral equation.

REFERENCES

[1] Jafari H. ,*A new general integral transform for solving integral equations*, Journal of Advanced Research,32(2021),133-138.

[2] Aboodh KS. *The new integral transform aboodh transform.* Global J Pure ApplMathe 2013;9(1):35–43.

[3] G. Adomian, Solving Frontier Problems of Physics: The Decomposition Method, Kluwer Acad. Publ., Boston, 1994.

[4] A.M. Wazwaz, A new technique for calculating Adomian polynomials for nonlinear polynomials, Appl. Math. Compt., 111 (2002), 33-51.

[5] S.A. Khuri, A Laplace decomposition algorithm applied to a class of nonlinear differential

equations, Journal of Applied Mathematics, 1 (2001),141–155.

[6] S. Abbasbandy, *Improving Newton–Raphson method for nonlinear equations by modified Adomian decomposition method*, Applied Mathematics and Computation 145 (2003) 887–893.