Study of Laminar Flow between Parallel Plates by Using KAJ Integral Transform

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Abstract- In 2022 a new integral transform called the "Kuffi- Abbas- Jawad" KAJ integral transform has been proposed and applied to solve ordinary differential equations. The proposed KAJ integral transform is a modification of the well-known (Sadiq-Emad-Eman) SEE integral transform. It is a novel method that could be used to solve ordinary differential equations. In this paper we use KAJ transform to study the laminar flow between parallel plates.

Indexed Terms- KAJ transform, Integral transform, Laminar flow, Parallel plates.

I. INTRODUCTION

Recently, integral transforms are one of the most useful and simple mathematical technique for obtaining the solutions of advance problems occurred in many fields like science, Engineering, technology, commerce and economics. To provide exact solution of problem without lengthy calculations is the important feature of integral transforms.

Due to this important feature of the integral transforms many researchers are attracted to this field and are engaged in introducing various integral transforms. Recently, Kushare and Patil [1] introduced new integral transform called as Kushare transform for solving differential equations in time domain. Further, Savita Khakale and Dinkar Patil [2] introduced Soham transform in November 2021. As researchers are interested in introducing the new integral transforms at the same time they are also interested in applying the transforms to various fields, various equations in different domain. In January 2022, Patil et al [3, 4, 12, 31, 32, 34, 37, 40] used Kushare transform solving different problems.

D.P. Patil [5, 6, 8] used Sawi transform for solving various types of problems. Laplace transforms and Shehu transforms are used in chemical sciences by Patil [7]. Using Mahgoub transform, parabolic boundary value problems are solved by D .P. Patil [9]

D.P. Patil [10] used double Laplace and double Sumudu transforms to obtain the solution of wave equation. Further Dr. Patil [11] also obtained dualities between double integral transforms. D. P. Patil [13] solved boundary value problems of the system of ordinary differential equations by using Aboodh and Mahgoub transforms. Double Mahgoub transformed is used by Patil [14] to solve parabolic boundary value problems.

Laplace, Sumudu , Aboodh , Elazki and Mahagoub transforms are compared and used it for solving boundary value problems by Dinkar Patil [15]. Futher, Patil with Tile and Shinde [17] used Anuj transform and solved Volterra integral equations for first kind. Rathi sisters and D. P. Patil [18] solved system of differential equations by using Soham transform. Vispute, Jadhav and Patil [19] used Emad Sara transform for solving telegraph equation. Kandalkar, Zankar and Patil [20] evaluate the improper integrals by using general integral transform of error function. Dinkar Patil, Prerana Thakare and Prajakta Patil [21] obtained the solution of parabolic boundary value problems by using double general integral transform.

D. P. Patil et al [22, 23, 25] used various integral transform to obtain the solution of Newton's law of cooling. Dinkar Patil et al [24, 28, 29, 30, 39, 46, 47] used integral transforms for handling growth and Decay problems, D. P. Patil et al [26] used Emad-Falih transform for general solution of telegraph equation. Dinkar Patil et al [27] introduced double kushare transform. Recently, Wagh sisters and Patil used Soham [33] transform in chemical Sciences. Raundal and Patil [35] used double general integral transform for solving boundary value problems in partial differential equations. Rahane, Derle and Patil [36] developed generalized double rangaig integral transform.

Kandekar et al [38] used new general integral equation to solve Abel's integral equations. Thakare and Patil [41] used general integral transform for solving mathematical models from health sciences. Rathi sisters used Soham transform for analysis of impulsive response of Mechanical and Electrical oscillators with Patil [42]. Patil [43, 45] used KKAT transform for solving growth and decay problems and Newton's law of cooling. Suryawanshi et al [16, 44] used Soham transform for solving volterra integral equations and mathematical models occurring in health science and biotechnology.

This paper is organized as follows. Introduction is in first section. Second section is for preliminary concepts. Third section is devoted to application of KAJ integral transform to study laminar flow between parallel plates.

II. PRELIMINARY

In this section we state some preliminary concepts required to study laminar flow between parallel plates.

KAJ Transform Definition

KAJ Transform (denoted by S_m) as a modification on the SEE integral transform, is given by: $S_m\{f(t) = K(v) = \frac{1}{v^n} \int_{t=0}^{\infty} f(\frac{t}{v})e^{-t}dt$, n∈ Z, 0< $l_1 \le v \le l_2$, where l_1 and l_2 are either finite or infinite.

KAJ Transform of Derivatives: Let f(t) be a function whose KAJ transform is K(v) then

$S_m\{f'(t)\} = [vK(v) - \frac{v}{v^n} f(0)]$.

$S_m\{f''(t)\} = \frac{1}{v^n}[-v^2 f(0) - vf'(0)] + v^2 K(v)$.

III. USE OF KAJ TRANSFORM IN LAMINAR FLOW BETWEEN PARALLEL PLATES

Consider a steady and uniform laminar flow of the viscous fluid between the two parallel plates situated at a perpendicular distance L. Let the distance in which the fluid is flowing be represented by x and the distance which is normal to the flow of fluid and parallel to the plane of paper be represented by y such that the lower plate is situated at $y = 0$ and the upper plate is situated at $y = L$.

The flow characteristics equation of motion of a viscous fluid is given by

$\mu\ddot{U}(y) = \frac{dp}{dx}$ ....(1)

We will study the laminar flow of viscous fluid on the basis of following assumptions:[48]

i) The flow is steady and incompressible and the properties of the fluid do not vary in the direction normal to the direction of flow of fluid.
ii) There are no end effects of the surfaces on the viscous fluid.
iii) There is no relative velocity of the fluid with respect to the surface of the plates.
iv) There is a uniform effective pressure gradient in the direction of flow of fluid i.e. $\frac{dp}{dx}$ is a constant in x-direction.

Now taking KAJ Transform of equation (1) , we get

$$S_m[\mu\ddot{U}(y)] = \frac{dp}{dx} S_m\{1\}$$

This equation results,

$\mu[\frac{1}{v^n}[-v^2 U(0) - v\dot{U}(0)] + v^2 K(v)] = \frac{dp}{dx}[\frac{1}{v^{n+1}}]$ ………..(2)

A. For Laminar Flow Between Stationary (Fixed) Parallel Plates

Considering the flow of fluid between two parallel fixed plates, we can write the relevant boundary conditions as given below:

At $y = 0$ and $y = L, U = 0$.

Applying boundary condition: $U(0) = 0$, equation (2) becomes,[48]

$\frac{1}{v^n}[-v\dot{U}(0)] + v^2 K(v) = \frac{1}{\mu}\frac{dp}{dx}\frac{1}{v^{n+1}}$ …………..(3)

In this equation, $\dot{U}(0)$ is some constant so let us substitute $\dot{U}(0)=\varepsilon$. Also, since $\frac{dp}{dx}$ is uniform, therefore, put $\frac{dp}{dx} = -\emptyset$, where $\emptyset$ is a constant and negative sign indicates that the pressure of fluid decreases in the direction of flow of fluid.

Equation (3) becomes,

$K(v) = \frac{1}{v^{n+1}}\ \varepsilon - \frac{\emptyset}{\mu}\frac{1}{v^{n+3}}$ …………(4)

Taking inverse KAJ Transform of equation (4), we will get,

$U(y) = \ \varepsilon y - \frac{\emptyset}{2\mu}y^2$ …………(5)

1) Determination of the constant ε :

To find the value of constant ε , applying boundary condition: $U(L) = 0$, equation (5) becomes,

$\varepsilon = \frac{\emptyset}{2\mu}L$ ….(6)

Substitute the value of ε from equation (6) in equation (5) we get,

$U(y) = \frac{\emptyset}{2\mu}[Ly - y^2]$ ….(7)

Differentiating equation (7) w.r.t. y we get,

$\dot{U}(y) \ = \frac{\emptyset}{2\mu}[L - 2y]$ ….(8)

For maximum velocity $\dot{U}(y)$ =0,

This results ,

$y = \frac{L}{2}$ ….(9)

Put the value of y from equation (9) in equation (7) we get,

$U_{max} \ = \ \frac{\emptyset}{8\mu}L^2$ ….(10)

The shear stress distribution is determined by the application of Newton's law of viscosity as

$\tau(y) = \mu\dot{U}\ (y)$

Using equation (8) we get,

$\tau(y) = \frac{\emptyset}{2}$[L-2y] ……(11)

At $y = \frac{L}{2}$ i.e. at the mid of the fixed parallel plates, $\tau\left(\frac{L}{2}\right) = \frac{\emptyset}{2}\left[L - 2\left(\frac{L}{2}\right)\right] = 0$

There is no shear shear stress even whn there is constant pressure gtadient.

At y=0 i.e. at the surface of fixed plate, $\tau(0) = \frac{\emptyset}{2}$L.

At y=L i.e. at the surface of the upper fixed plate, $\tau(L) = -\frac{\emptyset}{2}$L.

For a particular case, when $\emptyset = 0$, $\tau(y) = 0$ i.e. these is no shear stress between the fixed parallel plates if there is no pressure gradient.

B. For Laminar flow between parallel plates having relative motion

Considering the flow of fluid between the parallel flat plates such that the lower plate is fixed at y = 0 and upper plate is moving uniformly with velocity U_0 relative to the lower fixed plate in the direction of flow of fluid, we can write the relevant boundary conditions below:

At y = 0, U = 0 and y = L, U = U_0

Applying boundary conditions : U(0) =0, equation (2) becomes,

$\frac{1}{v^n}[-v\dot{U}\ (0)] + v^2K(v) = \frac{1}{\mu}\frac{dp}{dx}\ \frac{1}{v^{n+1}}$ …..(12)

In this equation , $\dot{U}$ (0)= δ = constant. And $\frac{dp}{dx}$ = constant.

Equation (12) becomes,

$\boldsymbol{K(v)} = \frac{\boldsymbol{\delta}}{\boldsymbol{v^{n+1}}} - \frac{\boldsymbol{1}}{\boldsymbol{v^{n+3}}}\frac{\emptyset}{\boldsymbol{\mu}}$ ……(13)

Taking inverse KAJ Transform of equation (13) we get,

$\boldsymbol{U(y) = \delta y - \frac{\emptyset}{2\mu}y^2}$ …..(14)

1) Determination of constant $\boldsymbol{\delta}$

To find the value of constant $\boldsymbol{\delta}$, apply boundary condition: $\boldsymbol{U(L) = U_0}$

equation (14) becomes,

$\boldsymbol{U_0 = \delta L - \frac{\emptyset}{2\mu}L^2}$

Solving for $\boldsymbol{\delta}$, we get,

$\boldsymbol{\delta = \frac{U_0}{L} + \frac{\emptyset}{2\mu}L}$ …..(15)

Substituting the value of $\boldsymbol{\delta}$ from equation (15) in equation (14)

$\boldsymbol{U(y) = \frac{U_0}{L}y + \frac{\emptyset}{2\mu}[Ly - y^2]}$ ……(16)

This equation (16) confirms that the velocity distribution is parabolic with minimum at the lower fixed plate.

Differentiating equation (16) w.r.t. y, we get,

$\dot{\boldsymbol{U}}$ (y)= $\boldsymbol{\frac{U_0}{L} + \frac{\emptyset}{2\mu}[L - 2y]}$ …….(17)

For maximum velocity $\dot{\boldsymbol{U}}$ (y)= 0.

This results,

$\boldsymbol{y = \frac{\mu}{\emptyset}\frac{U_0}{L} + \frac{L}{2}}$ …….(18)

Put the value of y from equation (18) in equation (16) we get,

$\boldsymbol{U_{max}} = \frac{\mu}{\emptyset}\frac{\boldsymbol{U_0}^2}{\boldsymbol{L}^2} + \frac{\boldsymbol{U_0}}{\boldsymbol{2}} + \frac{\emptyset}{\boldsymbol{8\mu}}\boldsymbol{L}^2$ …..(19)

The shear stress distribution is determined by the application of Newton's law if viscosity as

$\boldsymbol{\tau(y) = \mu\dot{U}\ (y)}$

Using equation (17) , we get,

$\boldsymbol{\tau(y)} = \frac{\mu U_0}{L} + \frac{\emptyset}{2}[L - 2y]$ …..(20)

At $y = \frac{L}{2}$, i.e. at the mid of the flow passage, $\tau\left(\frac{L}{2}\right) = \frac{\mu U_0}{L}$

At $y = 0$, i.e. at the surface of the lower plate, $\tau(0) = \frac{\mu U_0}{L} + \frac{\emptyset}{2}L$

At $y = L$, i.e. at the surface of the upper plate, $\tau(y) = \frac{\mu U_0}{L} - \frac{\emptyset}{2}L$

For a particular case when $\emptyset = 0, \tau(y) = \frac{\mu U_0}{L}$ i.e. the shear stress between the plates is not zero and moving a constant value even if there is no pressure gradient.

CONCLUSION

In this paper, we have studies the unidirectional Laminar flow between stationary parallel plates and have successfully obtained the velocity and shear stress distribution of a unidirectional Laminar flow between stationary parallel plates as well as between parallel plates having a relative motion by solving the differential equation describing the flow characteristics of a viscous fluid via KAJ integral transform. Thus KAJ integral transform has presented a powerful tool for obtaining the solution of the differential equation representing flow characteristics without finding the general solution.

1) Research paper of study of laminar flow between parallel plates via Gupta Integral Transform.

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